Learning Maths with a Tangible User Interface: Lessons Learned through Participatory Design with Children with Visual Impairments and Their Educators

Ana Cristina Pires[a,*], Ewelina Bakala[b], Fernando Gonzalez-Perilli[c], Gustavo Sansone[d], Bruno Fleischer[c], Sebastian Marichal[e], Tiago Guerreiro[a]

[a] *LASIGE, Faculdade de Ciências, Universidade de Lisboa, Portugal*
[b] *INCO, Facultad de Ingeniería, Universidad de la República, Uruguay*
[c] *SAMLA, Facultad de Información y Comunicación, Universidad de la República, Uruguay*
[d] *Facultad de Arquitectura Diseño y Urbanismo, Universidad de la República, Uruguay*
[e] *Universitat Pompeu Fabra, Barcelona, Spain*

Abstract

Through a set of participatory design (PD) sessions with children with visual impairments and their educators, we understood current practices in maths teaching, and designed a novel system to support learning for this particular educational context. Sixteen children were engaged in 19 PD sessions to develop tangibles and auditory stimuli to represent numbers, and to explore activities to use through a tangible user interface. We describe the context and lessons learned along the PD process with children and educators, and their implications on the design. Two main outputs were derived: iCETA, a multimodal tangible user interface that allows the use of tangible blocks to represent numbers; and, Logarín, an audiogame designed for iCETA that enables mathematical training. We explored the use of iCETA and Logarín during 15 sessions with 11 children with visual impairments. Results indicate that playing Logarín is engaging and capable of promoting their mathematical abilities. This research supports evidence that PD is successful in bringing children and other stakeholders together to design a solution that fits children's needs and promises educational impact.

Keywords: Maths, Learning, Children with Visual Impairments, Tangibles, Participatory Design

*Corresponding author: acdpires@fc.ul.pt

Figure 1: iCETA. Left: iCETA: headphones, computer, mirror in the camera, tangible blocks and working area on top of the keyboard. Middle and Right: Children playing with iCETA using the blocks to solve auditory addition composition tasks.

1. Introduction

A common way to introduce mathematical concepts at early ages is through the use of manipulatives, objects that serve as external representations of numbers and quantities. Children perform mathematical operations (e.g., counting, grouping and dividing) by interacting with objects, which reduces cognitive load and frees up working memory resources to focus on the mathematical operations [1]. In the context of visual impairments, the interaction with manipulatives that emphasizes a mathematical content, may contribute to a better understanding of mathematical operations, and to develop sophisticated strategies for active touch [2]. Active touch counting strategies involve: 1) preliminary scanning - scan the elements before counting; 2) counting organisation - organise the counting process by following given features like dot lines or circles; and 3) partitioning - strategies to keep track of elements already counted [3].

The inclusion of auditory information into active touch strategies also represents an opportunity to reinforce maths understanding [4]. Blind children rely on auditory representations to count elements [5, 6, 4], and with time and experience, they develop extraordinary capabilities in auditory perception, as in counting the number of beats in a rhythm [4]. Thus, auditory representations of numbers can be a useful resource when teaching maths to children with visual impairments, for example, by creating structures such as "part-whole" relationships to facilitate the understanding that numbers are also compositions of other numbers [7]. Auditory stimuli can be used together with other types of sensory information to provide multimodal feedback taking further advantage of children's sensory systems. In this context, the use of multimodal interfaces - that afford embodied, spatial, haptic and auditory interactions with objects - can be particularly relevant to support active learning strategies [4].

In this paper, we describe the process of designing a low-cost and accessible tangible user interface (TUI) (Figure 1) and the lessons learned from 9 focus groups with stakeholders and 19 participatory design (PD) sessions with children with visual impairments. In Phase I, we led interviews and focus groups with stakeholders and educators to understand children's educational context, math practices and technology usage in Uruguay. In Phase II, we aimed to identify the characteristics that tangibles should have to be interpreted as number representations in iterative cycles of prototyping and testing with children. In Phase III, we developed a prototype of the system with a math activity to observe how children interacted with the setup in iterative cycles of prototyping and testing to define new requirements. Then, in Phase IV we co-developed the final prototype of the system - iCETA - and the math audiogame - Logarín - with experienced researchers on math cognition (members of our team) and experts of sound design. And finally, we evaluated the use of our TUI in 15 sessions at children's schools, in a 3-week deployment and the results showed that children had an engaging experience that may have promoted their mathematical abilities.

2. Related Work

We reviewed the literature on three topics central to our contributions: learning maths, TUIs for learning (including maths) and PD with children with visual impairments.

2.1. Learning maths

At the beginning of formal education, children typically use concrete material as manipulatives, cusinaire rods, abacus or counting beads to learn mathematical relations and understand abstract concepts. Manipulatives are frequently used in classes to support cardinality acquisition and basic operations such as subtraction and addition, fractions and place value [8, 9]. Such concrete experiences enables to physically sense abstract numerical relations [10] and serve as a scaffolding for the mathematical properties children gradually have to learn [11].

Abacus and Montessori-like manipulatives are frequently used by educators of children with visual impairments to assist in the practice of addition and subtraction operations [12, 13]. The use of Abacus allows counting and to separate or join beads in clearly differentiated spaces whereas Montessori-like manipulatives deploy a length-quantity relationship enabling children to associate size and quantity. This kind of materials affords children to haptically explore size and quantity and reflect on their own actions [14] while being instructed by teachers, which ease and strength their learning experience [14, 15].

After children learned braille, they will be able to take advantage of the Nemeth Braille Code [16] as well, a variation of the former, specially designed for mathematical purposes. However, this abstract symbolic representations of numbers is not the best tool for the first approach to numerical facts since it requires some previous numerical knowledge.

2.2. Accessible TUIs for learning

TUIs present multiple opportunities to support the learning of children with visual impairments but there has been limited research in exploring such interfaces, and none, to our knowledge, specifically for mathematical operations with numbers. We summarize research using TUIs to promote learning in the three most explored domains: Braille, computational thinking and related-maths.

Electronic Braille Blocks [17] is a set of tangible blocks with embossed Braille letters for young blind children to learn and reinforce Braille letter recognition by providing auditory feedback. Similarly, Gadiraju et al. [18] developed Braille-Blocks - a set of tangible blocks and pegs that children use to compose words using the blocks.

Recently, some research has been focused on creating systems to enable the learning of Computational Thinking skills. In StoryBlocks [19], children use a set of blocks to create audio stories enabling programming conceptualisation. Thieme et al. [20] explored the creation of digital music to train computational thinking using interconnected multimodal beads. In the same effort to support new ways to foster children with visual impairments computational thinking, recent studies [21, 22, 23, 24] started to explore, jointly with educators, parents and children, the use of tangibles to spatially move a robot in a tangible map.

Lastly, Jafri et al. [25, 26] developed a TUI and several educational maths-related activities, based on the distribution of three-dimensional geometric figures, where a computer provides immediate audio feedback regarding shapes and spatial relations. Ruhmann et al. [27] developed an Android application with a tangible appcessory to enable users with visual impairments to explore simple geometric forms displayed on a tablet through sound and vibrotactile feedback. Another multimodal TUI, the Trackable Interactive Multimodal Manipulatives [28] enables children with visual impairments to interact with graphical representations on a multitouch surface with auditory feedback. McGookin et al. [29] created the Tangible Graph Builder, a tabletop TUI system to allows users to browse and construct both line and bar graphs non-visually. Similarly, Manshad et al. [30] developed a system based on MICOO (interactive multimodal cubes for object orientation) and on an interactive table, where children can create and modify diagrams and graphs while being guided by auditory feedback. Such work was further extended to provide more diverse feedback, such as speech, sound/music, vibration and force feedback [31]. However, research on accessible TUIs for learning basic maths operations is lacking. Soto et al. [32] conceptualized using tangible magnetic numbers to learn maths, but no further development was made. We sought to bridge this gap by developing a multimodal and accessible TUI to support basic math learning.

2.3. Participatory design (PD)

There has been a growing line of research exploring new ways to include children with visual impairments in participatory or co-design sessions along with their sighted peers by providing a multiplicity and diversity of sensory elements [33, 34, 35]. Studies have been exploring the use of Voice User Interfaces at schools [36], robots [37, 21], therapeutic video games [38], accessible movement-based games and co-located games [39, 33], and other multisensory technologies [40, 41, 42, 43, 44]. For instance, Brulé and Bailly [41] developed geo-technologies that support non-visual sensory information about locations or geography by leading a PD approach with children. McElligott and van Leeuwen [42] provided a series of auditory and tactile elements to engage children with visual impairments as co-designers in the development of toys and sound tools. Another PD study developed MapSense, a multisensory system that allows children to use their touch, smell and taste to interact [44]. Although researchers are now more prone to include children with special educational needs in the design process, we still need more research advocated to specifically include children with visual impairments. In our study, we aimed to include stakeholders and children with visual impairments in a PD process to develop a technology that makes sense to the community and fits their needs.

3. Design of iCETA

This study aimed to develop an accessible low-cost tangible interface and computer game to promote maths learning while including children with visual impairments and stakeholders into the PD process. Four research questions motivated our work:

- RQ1: What are the current educational contexts, maths practices and technology usage of children with visual impairments in Uruguay?
- RQ2: What characteristics should the tangibles have in order to be interpreted as number representations?

- RQ3: How to design the TUI in the context of maths activities?
- RQ4: Do the developed system and audiogame support maths training?

To answer our research questions, we divided the design, development and evaluation process into four phases.

3.1. Recruitment of participants, data collection and analyses

Our research protocol was approved by the Local Research Ethics Committee of the Faculty of Psychology in Uruguay and is in accordance with the 2008 Helsinki Declaration. The research was authorized by the schools' directors and by the Public Education National Administration who oversaw the entire process. Stakeholders and parents/legal tutors signed consent forms to participate or to allow children to participate in this study. All participants assented to participate and were aware of their freedom to quit anytime during the research.

We followed a PD approach considering "children as informants" [45, 46] where children test raw prototypes, giving insights about their functionalities, usability, and expectations to further fuel the iterative design process. Through all the design phases, we observed and video recorded participants and collected field notes. After each encounter, two researchers discussed their notes and observations and finally, one researcher annotated all the relevant outcomes from the session. The two researchers used thematic analysis [47] to analyse videos and field notes to understand the contextual conditions for the new design and develop prototypes focused on children's feedback. Whenever the researchers found a discrepancy or doubts in the analyses, they triangulated observations with educators and children's opinions.

Sixteen children from first grade at either school 1 or 2 (see Table 1) participated in Phase II, III, and IV. Some participated more frequently than others as some children did not attend school regularly.

Table 1: Children involved in the PD sessions. Table indicates in which Phase did children participated, their age, sex, visual impairment, and comorbidities.

Phase	Age	Sex	School	Visual Impairment	Comorbidity
II, III, IV	5y4m	M	2	Low-Vision	-
II, III, IV	6y6m	M	2	Blind	-
II, III, IV	6y9m	F	2	Low-Vision	Attention Deficit Hyperactivity Disorder
II, III, IV	7y8m	F	2	Low-Vision	-
II, III, IV	7y9m	F	2	Low-Vision	-
II, III, IV	8y0m	F	1	Low-Vision	-
II, III, IV	8y4m	M	2	Low-Vision	-
II, III, IV	8y10m	M	1	Low-Vision	-
II, III, IV	9y8m	M	1	Blind	Global Development Delay
II, III	10m11m	M	1	Blind	-
II, III	10m6m	F	1	Blind	Autism
II, III, IV	11y8m	F	1	Low-Vision	-
II, III	11m11m	M	1	Low-Vision	-
II, III	11m11m	M	1	Blind	Autism
II, III, IV	12y4m	M	1	Blind	-
II, III	12m4m	F	1	Blind	Global Development Delay

4. Phase 1: Educational Context, Maths Practices and Technology Usage

Our design process began with three focus groups with experts of different public educational services for children with visual impairments in Uruguay: National Blind Union, National Inspection of Special Education and Center of Resources. As a result of these sessions, we understood the educational, cultural and socio-economic contexts, current practices and public laws related to education of children with visual impairments. We then focused on the two existing institutions specialised in children with visual impairments in Uruguay to understand current practices in teaching and learning mathematics and also to understand the global learning process children are embedded in. We held four semi-structured interviews and two focus groups in each special education school (schools 1 and 2) with the following educational professionals: two school directors, three elementary teachers, one blind music teacher, one blind Braille teacher and one informatics teacher. We guided the session to obtain information about children's evolution and major difficulties in math learning, which materials and methodologies did educators use, which technology, and to also explore the possibility of using music to learn mathematics. At the end of the sessions, we presented an example of a TUI for maths learning to fuel further discussions around TUIs potential for the present research.

4.1. Findings

In this phase, we achieved a deeper understanding of children's needs and context. Getting to know the materials and tasks used in class inspired the development of iCETA and helped to identify mathematical contents that should be reinforced. We now present the main findings from the focus group sessions and interviews with stakeholders, and our field notes from observations *in-situ*.

4.1.1. Educational context and children prevalence in special schools

In Uruguay, children with visual impairments attend either inclusive schools with adapted and accessible learning materials or specialized schools for children with visual impairments - special schools. A great number of children with visual impairments attend special schools, often because they lacked earlier cognitive stimulation or they present some level of comorbidity - e.g., cerebral paralysis, autism, motor, cognitive impairments or learning difficulties - which makes learning more challenging. Due to this heterogeneity, they require special attention and a range of educational resources. It is common that children begin their education in special schools and later on, move to inclusive schools.

4.1.2. Learning maths with manipulatives

All educators referred that a great part of the children in special schools has serious difficulties in mathematics and struggle with the concept of number. The class methodology is mainly based on manipulation and practice, repetition and collaborative working. Children learn maths by using abacus and manipulatives, such as bottle caps, measuring tape, LEGO blocks, etc (Figure 2). Children understand one-to-one representations, but they are not familiar with one-to-many representations (in which a single object can represent a group of n values). Educators pointed out the benefits of counting forward and backwards with the use of concrete materials, which help children understand the number concept. Children group manipulatives to solve additive composition and decomposition activities, and later on, pass to complex activities focused on division and multiplication of numbers. When brainstorming about the ideal manipulatives to learn maths, educators remarked some relevant features: manipulatives should not be too small, have different colours and sizes, and incorporate the number/value in Braille. Educators referred to the benefits of designing large blocks to include children with motor disabilities. They also pointed out that blocks could have notches on one side, similar to a measuring tape (Figure 1b), and on the other, Braille, so the children can relate quantity (of notches) to numbers (represented in Braille).

Educators stressed out the need to limit the area where children manipulate objects because they could easily lose localisation of the material - which may impair access to all the information in working memory.

Based on the collected information, we decided to develop activities to work on the concept of number, an important basic concept that, when acquired, allows children to learn more advanced mathematical contents. We decided to focus on forward and backward counting and additive composition and decomposition tasks with manipulatives.

4.1.3. Learning Braille

Schools give special attention to train braille and children have a dedicated teacher to braille since 5-6 years old. Educators emphasized that it is highly beneficial to read and write braille (including braille numbers) as it is a tool important for their autonomous and independent learning. In agreement, we decided to include braille numbers in the manipulatives.

4.1.4. Music

Educators reported that children enjoy music and exercise different skills related to music and bodily expression. In music classes, they practice reading and writing music in braille, building musical instruments and using electronic

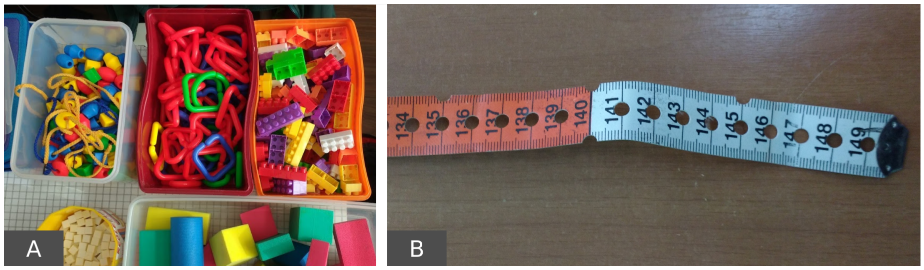


Figure 2: A) Different objects used at school to train maths. B) Accessible measuring tape for children with visual impairments. To indicate 1 cm it has a marker on the middle, to each 5 cm it has a side marker and to each 10 cm it has a double side marker.

> Two photos. At the left it shows different objects to train maths, such as LEGO bricks, geometrical solids, little cubes, rings and wire to connect different spheres. At the right a photo of an accessible measuring tape.

musical devices. The music teacher emphasized that these children have a strong auditory capacity. We foresaw this as an opportunity to work on mathematical concepts by exploring the relationship between numbers and sounds. Even though music was not explored and used as a tool to learn and train numerical concepts in classes, the teacher suggested that it could be very relevant to support the understanding and perception of quantity and numbers.

4.1.5. Using Computers

In Uruguay, each child with visual impairments receives a personal computer with NVDA screen reader[1]. In informatics classes, children perform activities in word processors and on the web. They learn how to perform a Google search, to use social networks and play educational digital tasks, e.g., Musibraille, Cantaletras, Mekanta, MecaNet. One of the games "*La Pulga Leocadia*" trains basic numerical and spatial concepts as well as orientation. Only one game was considered by the teachers as a successful gaming experience; "*Blind Legend*"[2]. Is an action-adventure video game without video/ graphics with binaural sound for

[1] https://www.nvaccess.org/download/
[2] http://www.ablindlegend.com/en/home-2/

an immersive and enjoyable experience. Educators also remarked that games with background music were not welcomed as children get stunned and have difficulty distinguishing other relevant sounds of the game.

Children enjoy the use of headphones so they can be focused on their activities while avoiding listening to the screen readers of other pupils. Given the frequent use of headphones and the poor offer of entertaining activities, we decided to explore gamification elements and binaural sound to make the learning content more attractive.

Educators also emphasized that to facilitate the interaction with the audio-game, children value to first experience the task (have an embodied experience) and then play the computer game so that the game dynamics could be anticipated by a real-life experience. For example, if they will play a game about counting apples, they should grab some apples and count them first.

4.1.6. TUIs as a potential tool for maths learning

At the end of each focus group, we showed an example of a TUI [48] to learn mathematics designed for sighted children to further fuel discussions and visualise possible benefits. Educators enjoyed the tool and saw opportunities to use this type of systems in their activities. Educators endorsed the possibility to manipulate blocks while having at the same time auditory feedback as very beneficial for the process of learning maths. They envisioned children using manipulatives with real-time feedback so children would be able to practice maths with manipulatives independently and autonomously.

5. Phase II: Tangibles

Based on the previous findings, we defined our end users to be children from the first grades of special schools who needed early maths training, and we started to explore tangibles to represent numbers from 1 to 5. We began initial explorations with the same blocks as in [49] to answer our second research question:"*What characteristics should the tangibles have in order to be interpreted as number representations?*". Blocks varied in length and colour: the block with 1 unit represents the number "1"; and so forth until 5. We used them as a starting point for a series of iterations to inform the design of the final manipulatives. We enhanced the blocks with tactile patterns (textures and markers) to help distinguish the different blocks [50, 51, 19]. Here, for simplicity, we refer to the four main relevant prototypes developed and their features (Figure 3).

We started by designing bar shaped blocks inspired by the marks used in an accessible measuring tape (Figure 2B) that children used at school: blocks A with hollow markers to identify the units; and blocks B with hollow markers, and raised markers to identify the middle part of each unit (Figure 3A and B). In the next prototypes (C and D), we changed the form of the blocks to a row shaped composed of circles, and added braille. In block C we placed braille at the top of the first circular unit and added different textures to each block. Blocks D had no texture and had the double height to keep the standard size of braille located at the side of the first block unit.

5.1. Workshops with children

We carried out 8 workshops with the same structure: introduction, showcase, and a brief discussion. Each workshop gave us insights to improve the next prototype, following an iterative design process. In the first two workshops, one in each school, aimed to warm up and familiarize children with researchers and to assess their general understanding of manipulatives as numbers (by asking children to show us objects that represented numbers and explaining their meaning). In the following workshops, we playfully engaged children to characterize and categorize our blocks, explore its features, watch for differences and similarities, and to group blocks by applying a common rule. In the third, forth and fifth workshops with four children with low vision and one blind child at school 2, we explored children's interaction with the bar shaped blocks. In the sixth, seventh and eighth workshops we tested the prototypes C and D at school 1 with a total of eight children with low-vision and four blind children.

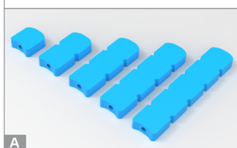
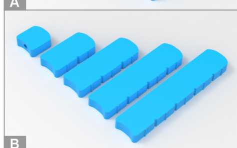
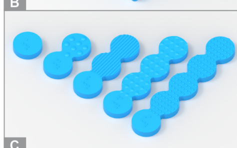
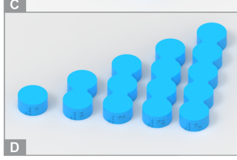
Blocks	Identification of units	Braille	Textures
	- Hollow markers	-----	-----
	- Hollow markers - Raised markers	-----	-----
	- Circular forms.	- At the top of the first unit.	- One for each block type.
	- Circular forms.	- At the side of the first unit of the block. To keep braille standar size we ingreased block's height.	-----

Figure 3: The main different type of blocks (A, B, C and D) used in the workshop sessions with children with their main features. Blocks represent numbers from 1 to 5.

Four types of blocks representing numbers from 1 to 5. The identification of units in Blocks A is represented by hollow markers. Blocks B have hollow markers,and raised markers to identify units. Blocks have circular forms to represent each unit and braille at the top of the first unit. Blocks D are also circular and have braille at the side of the first unit and it has the double height with relation to the previous blocks.

5.2. Findings

These sessions informed us about the characteristics that the tangibles should have to be interpreted as number representations. Lessons learned from all the workshops are presented together for simplicity (see Table 2).

5.2.1. Do not overload manipulatives with tactile cues

We found that almost all children had difficulty relating the bar shaped blocks (Figure 3) to numbers concepts. Both types of markers in these blocks were detected only by one blind student. Neither the use of a single mark (A) or double mark (B) led to the understanding that such features indicate units that can be counted up. The use of too many tactile cues providing the same type of information made it difficult to ascertain which information to rely on. Our results are in agreement with the findings of Sabuncuoglu et al. [52] that too many tactile cues may overwhelm and confuse users. To communicate the value of the block, we decided to iterate over the form of the blocks and remove the confusing markers.

5.2.2. Manipulatives with pronounced units were strongly associated with numbers

We changed the form of the tangibles to improve children's recognition of the number of units the block was composed of. By giving them a circular form we diminished the intersection between units and blocks' units became more pronounced. When we tested blocks composed of circular units - C and D - children easily counted the number of units and associated them with the number the blocks represented. This design worked well and the use of other marks to facilitate units' detection were unnecessary.

5.2.3. Textures were not associated with numbers, contrary to blocks' size and amount of units

The use of different textures to represent numbers (C) was unsuccessful. To recognise blocks as numbers, children relied on the size or/and quantity of circles/units. We studied several textures but children had difficulties in associating such a feature with number representation. Textures were a confusing cue and do not communicate the required characteristic. Because there is no natural mapping between textures and numbers, children would need to associate and memorize this arbitrary association, making the learning processing more cognitively demanding.

5.2.4. Braille at the top of the first unit disrupt the counting of block's units

We tested if children perceived and understood braille inscriptions on the blocks that corresponded to the number each block represented. We designed blocks *C* to have the braille at the top of the first unit. However, we found that besides the fact that they understood the braille inscription, its position disrupted counting the units of the blocks. For instance, when manipulating the block "two" children would read the braille inscription in the first unit and then move the finger to the next unit and start to count "one" (when in fact they were already with the finger at the second unit). Children did not start to count naturally at the first unit where braille inscription was located. This result is related to the fact that children are used to have braille in objects as a label, and thus, the position were the braille was located was not considered by them as a unit itself. To overcome this difficulty, we developed other types of blocks with the braille at the side of the block - blocks *D*. We increased the height of the blocks to keep the braille standard size. Children easily understood the location of the inscription and counted correctly the quantity of circular units that corresponded to the number that the block represented.

5.2.5. Provide affordances to place the manipulatives in the position in which braille is well-oriented

In the case of the blocks *D*, braille was not detected by the majority of children when its orientation was not correct; the block could be up or downwards (as also the number in braille). The correct orientation of the block was not clear enough so we improved the prototype to prevent children putting braille

Table 2: Main results of the PD process in Phase II: lessons learned, findings and context and the final design decision.

Lesson learned	Finding/Context	Design decision
1. Do not overload manipulatives with tactile cues	Too many tactile cues providing the same type of information (the value of the block) made it difficult to ascertain which information to rely on.	Reduce the amount of different tactile information that communicates the block's value. Block's units should be perceived and associated with the value of the block without the need of any extra mark.
2. Manipulatives with pronounced units were strongly associated with numbers	Circular units were easily identified and associated with numbers.	Units would have a circular form.
3. Textures were not associated with numbers, contrary to blocks' size and amount of units	Children associated the block's size and amount of units with numbers, but no textures.	Do not use textures, but size and amount of units to communicate the block's numerical value.
4. Braille at the top of the first unit disrupt the counting of block's units	Children did not start to count units at the first one if the braille inscription was located there.	Place braille on the side of the block, not on the top.
5. Provide affordances to place the manipulatives in the position in which braille numbers are well-oriented	In some cases children positioned blocks in a way that braille was upside down.	Use a tactile cue at the top of the block to facilitate it's correct orientation

upside down. We observed that braille was well oriented and well-read when using blocks C which had textures on the top; children naturally tended to put the textured side up. Following this observation and to avoid the use of textures that were confusing for children, we decided to add chamfered edges as an affordance to place the blocks correctly (see Figure 4).

5.3. Final Manipulatives: iCETA Blocks

Taking into account the previous findings, we developed a new set of tangible blocks to use in the next iteration of the PD process, based on prototype D, which was the most welcomed by children. We did not use textures and we did not add any other tactile clue to communicate the blocks' values. Braille was located at the side of the block and we added chamfered edges as an affordance

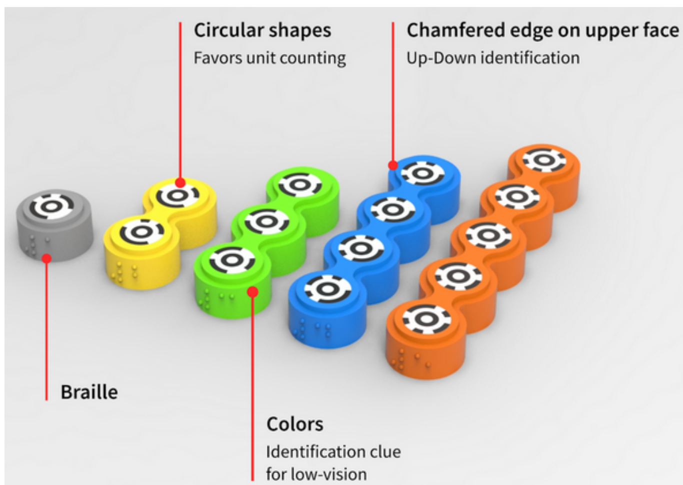


Figure 4: Final prototype of tangible blocks ranging from 1 to 5. Blocks differ in colour, size and braille numbers.

The Figure indicates the position of braille at the side of the first unit, the circular shapes of blocks to favour counting, the chamfered edge on upper face to identify block's up and down, and the colours to help children with low-vision to identify each number.

to place the block (and braille) in the correct position.

The final version of the blocks (see Figure 4) included TopCode [53] markers placed on the top to allow the computer vision system to detect and recognise each block. Similar to the fact that the blocks should be placed in the correct orientation to enable the reading of the number in braille, TopCodes also have to be placed correctly to ensure blocks' detection. We improved the prototype by highlighting the top face of the block by providing an upside protrusion - a chamfered edge.

It is important to note that each block needed just one TopCode to be identified by the system. However, considering children with low vision that also rely on vision to interact with the blocks, we decided to repeat TopCodes so that the number of TopCodes matches the number the block represented, and serves as an additional visual cue. In addition, it also increases detection probabilities. For instance, if the child covers a TopCode with his/her hand, there are other TopCodes that could be recognised by the system.

6. Phase III: TUI and maths activities

Once we developed the tangible blocks to represent numbers, we designed a prototype of a computer maths activity ideated by team members with experience in early maths cognition, and by invited experts in audio design. We were especially inspired by educators' opinions from Phase I, that highlighted the importance of training basic mathematical skills with manipulatives that could be combined with music and computer games.

The first prototype of the system (Figure 5) was composed by a tablet, tablet support, and mirror (similar to Pires et al.'s study [49]), the blocks designed in Phase II, and a simple computer task based on audio feedback. We handled the blocks' detection and feedback in real-time, using TopCodes on the blocks. The tablet detected the TopCodes by analysing a mirrored image of the camera's view.

We provided three different functional feedbacks in the audiogame allowing

Figure 5: First prototype of iCETA system composed of an Android tablet, wooden holder, 3D printed piece with a mirror (attached to the tablet) and tangible blocks.

Photo of the first prototype of iCETA

users to have audio feedback about their own actions [51] and to facilitate their interaction with the system (Figure 6): *detection sound*, *block sound* and *task sound*. *Detection sound* announces that a block was detected by the system and it is equal for all blocks - similar to a "beep". The *block sound* represents the numerical value of the block. It was represented by the sound of a piano. To indicate the number the child is asked to build, we created a *task sound*, that corresponded to the sound of drums. We also verbalized the number (verbal cue) at the beginning of each activity to indicate the number the child should build. Additionally, when children completed the task correctly, a positive verbal reward was played to provide children with positive feedback.

Task and *block sounds* (drums and piano, respectively) were reproduced in parallel (Figure 6). If the child has built a number bigger than expected - e.g., 2 is the number requested by the system, and the child put 4 -, the drums stop after 2 drums sounds and the piano continues to play (2 sounds more). The opposite happened when the number built by the child was smaller than

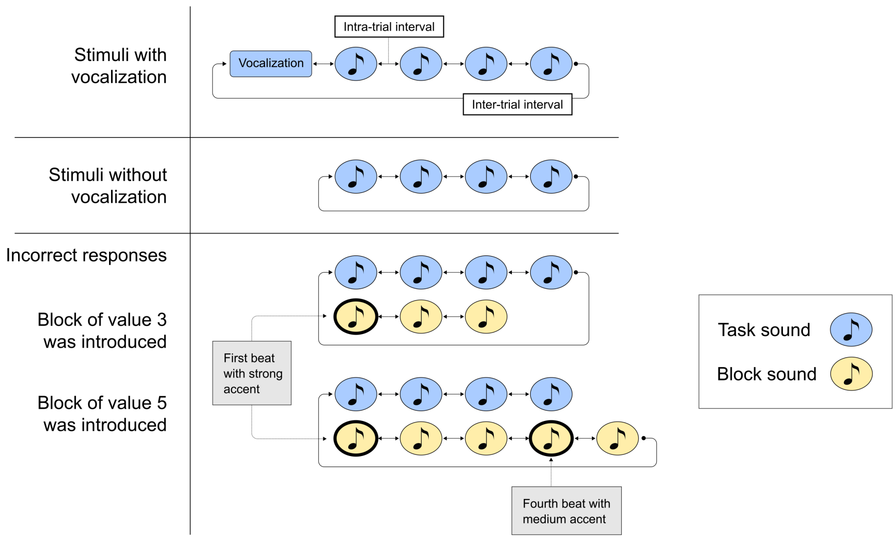


Figure 6: Scheme of sound feedback. Different beats series represent the required and submitted quantities.

This scheme shows that inter-trial intervals are always the same between beats. The first beat has a strong accent. In the case of the number 5, the fourth beat has a medium accent. For the stimuli with vocalization, the vocalization of the number is heard first, and then the corresponding beats of the number represented.

the number indicated by the system. The difference between the amount of drums and piano sounds was an indication that the answer was not correct and the difference in beats helped to understand exactly the difference between the number to build and the child's answer. We used this basic setting to start prototyping the system with children with visual impairments.

6.1. Workshops with children

We led eleven workshops with ten children with low vision and twelve blind children from both schools. Each workshop session had the aim to test the current prototype and to observe if children understood the interface and how to interact with it to solve the computer tasks. We started by familiarizing children with the blocks: categorize blocks, find differences and identify relationships, and find the braille. Before children play the game for 10-15 minutes we facilitate its understanding. Besides giving an oral explanation, we introduced an embodied example - for example: we knocked with our hands n times on the table and ask them to indicate which number we were representing. We explained that, for example, "knock-knock", means "one-two" and in the game they have to put the block of value 2 or two blocks of value 1. Besides, to ease the interaction with the system it is also important to understand how the system works and the logic of block's detection. To strengthen the "*how iCETA works*" mental model, we presented a tray with the blocks covered by the researcher's hand and asked the child to count how many blocks were in the tray. Failing to count the blocks (made inaccessible by the hand of the researcher covering the blocks), the child could understand the analogy that if he/she put the hand on the top of the blocks when playing, the system cannot detect the blocks. While interacting with the game we observed children's understanding of a) the relation between block's size and quantity, beats and quantity, and blocks and audio; b) the area in the table where blocks are detected by the camera; c) children's attention and engagement during the task.

6.2. Findings

Through a set of PD sessions, we gathered insights about the interaction between children, blocks, and the maths activity.

6.2.1. Help to locate and identify blocks

We observed that while children with low vision did not present any particular difficulty with the setup, blind children had sometimes difficulties interacting with the blocks. Their biggest challenge was to keep blocks accessible in working memory so they knew where they were located at anytime. Children held the block in their hands and counted its units to know which number it represents. The problem appeared when they put the block back on the table as, later on, it was difficult to know its value without the need to count each unit again.

Similar difficulties related with the location of materials involved in the activity was noticed by other authors like Najjar et al. [54] and da Rocha et al. [51]. To solve this issue we decided to have a box in which blocks could be ordered by number. This would make working memory resources available to, for example, attend to the mathematical concepts. The box should allow to store and order the 5 different blocks to permit their rapid access and identification.

6.2.2. Facilitate auditory stimuli discrimination

Drums and piano sounds were used to communicate the number that the child should build (*task sound*) and the value of the detected blocks (*block sound*). We found that we needed to include sounds or instruments with more distinguishable timbres and binaural sounds to facilitate the distinction between the audio clues as it was challenging for some children.

6.2.3. Help grouping and counting the sounds

For some children it was hard to detect the end of one loop and the start of a new one, because in some cases, the intervals between sounds were too short to afford quantity detection. Children were very heterogeneous in their inter- and intra-trail duration preferences, so we decided to enable its customisation in the last workshops. We also observed that it was challenging to count longer series. Thus, we explored different strategies aiming to enhance beats discrimination, and to group the sounds considering earcon's guidelines [55, 56]. The audio loop must have an explanation in the narrative that supports its understanding. Because tutorials are considered important resources to understand or refresh the game dynamics [19], we decided to provide a tutorial to reinforce the counting process. We observed that, when we explained how to count the sounds by clapping our hands, children understand the idea of counting the sounds easier and they were able to respond how many times we clapped and place the blocks necessary to answer correctly. We decided to add a tutorial with clapping, where children could count the sounds and also clap their hands at the same time.

Table 3: Main results of the PD process in Phase III: lessons learned, findings and context and the final design decision.

Lesson learned	Finding/Context	Design decision
6. Help to locate and identify blocks	Children struggled to remember where the blocks were and which value they had.	Create a box where blocks could be easily identified and ordered by number.
7. Facilitate discrimination between auditory stimuli	Children had difficulty distinguishing between the auditory representation of the block value and of the number they were required to build.	Include sounds with distinguishable timbres and used binaural sounds (task sound in one channel and block sound in the other). Provide a tutorial to reinforce the difference between audio stimuli.
8. Help grouping and counting the sounds	Children had difficulty in perceiving the beginning and end of sequences of sounds that represented numbers. There were individual preferences related to the inter-trial and intra-trial duration.	Follow earcons design guidelines. Enable the customisation of inter-trail and intra-trial duration by children.
9. After the initial excitement with the system, a great part of the children started to disengage with the setup.	The basic task that we developed was not appealing enough to keep the children motivated to proceed. The majority of children were not engaged after solving 5-6 tasks.	Provide a narrative to immerse and motivate children to keep doing the tasks. Add diegetic sounds and positive feedback.

6.2.4. Prevent disengagement with the activity

In this Phase, the digital maths activity did not have a narrative. We observed that after the initial excitement with the system and especially with the blocks, most of the children started to disengage after solving 5-6 mathematical activities. For the final solution, we added a narrative, diegetic and non-diegetic sounds and dialogues, and more varied positive feedback as we noticed that children needed more motivation and engagement to proceed in the tasks.

7. PHASE IV: Final iCETA prototype and Logarín

With the insights gained from the previous phases, we created the iCETA system (Figure 7) that runs on a laptop delivered to children with visual impairments in Uruguay, as part of the "One Laptop Per Child" program.

We created a laser-cut wooden tray to place over the laptop keyboard to

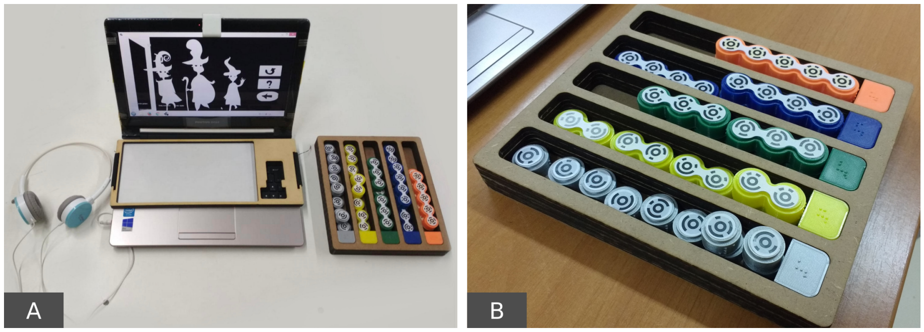


Figure 7: A) iCETA setup: headphones, computer, tangible blocks, 3D printed piece with a mirror, wooden tray that delimits the working area and a storage box. B) Box with the blocks representing values from 1 to 5.

It shows two photos. At the left the iCETA setup and at the right the storage box with the blocks ordered by number.

delimit the area in which the blocks could be detected by the camera - working area. We analysed the field of view of the laptop's camera to maximise the size of the tray to provide more space for the blocks while ensuring that all the blocks placed there were correctly detected. The tray allows the access to five keyboard keys - the four arrows and the Enter key - and was fixed to the laptop using an elastic band. We attached a 3D printed piece with a mirror to the laptop to enable the camera to capture the blocks placed on the wooden tray. Finally, we designed a storage box to store and organize blocks (Lesson Learned 6) with five separate grooves. Each groove has a braille reference with the same colour and braille number as the corresponding tangible block to facilitate its location and identification (Figure 7B).

7.1. Audiogame with math activities: Logarín

We co-designed the audiogame Logarín aimed to train maths with the tangible blocks together with invited experts in audio design.

7.1.1. Math activities

Interviews and the co-design workshops in Phase I and II helped to identify the mathematical level of children and the mathematical learning objectives for our TUI: forward and backward counting and additive composition and decomposition. In Phase III, we observed that the tangible system with the computer math activity may serve as a multimodal training and reinforcement of math learning. To this end, we developed math tasks to leverage the understanding of the commutative property of addition. Conversely, subtraction is not commutative and is harder to understand. To facilitate its understanding we introduced subtraction problems by solving related addition operations (e.g., 8 - 5 = 3, based on 5 + 3 = 8 [57]. Also, we also designed activities to train the associative property, i.e. to group small numbers to solve operations with larger numbers. We also explored additive composition and decomposition, n+1, n-1 counting, addition and subtraction patterns (2+2, 4+2, 6+2, 8-2, 6-2, 4-2, etc). The system represented the number by a series of sounds to be counted. To solve the activity, children use the blocks to compose numbers. The feedback was given as auditory beats corresponding to the number of the total of blocks being detected (see Figure 4). For more details see table at https://github.com/ewelinka/miceta/wiki/Levels.

7.1.2. Narrative and gamification

Driven by the lessons learned in the previous PD phases, we created a narrative to support a playful experience (see Lesson Learned 9). We co-designed the audiogame together with experts in audio design and created different funny sounds throughout the game to engage children in play and fun experiences. The narrative is the story of a young magician, Logarín, that is trying to become a great magician. Because Logarín is still learning, and he is very clumsy, he needs help to overcome the challenges presented at each level. Children have to guide Logarín's actions by introducing the "magical" blocks in the working area. The task difficulty increases as children progress in the game so they can have fun and meet challenges according to their pace and maths level. We created two different microworlds (i.e. small universes/environments) with 5 levels each. Each level starts with introductory audio that illustrates the new situation that Logarín has to deal with. The young magician is challenged by the tasks like repeating a bird song, knocking at the door in the correct order, making spells by adding the correct amount of drops for each ingredient, mixing the drinks in the correct sequence, etc. Each level has its own static background that illustrates the environment in which the story takes place. The background is a high contrast black and white image added as a decorative element for children with low vision and it has no animations to avoid distraction from the game's goal. After solving all tasks at the current level, the final audio is displayed to let the children know that they have successfully helped Logarín and completed the level. Because the game presents a novel interaction between tangibles, sound and mathematical concepts, we provided a tutorial with embodied examples (Lesson Learned 7), that can be accessed anytime to refresh the game dynamics. In the tutorial, children listen to the verbalised number and then the same number is represented by Logarín by clapping his hands.

7.1.3. Interstimulus intervals

Through several testing with children in Phase III, we decided to set 2 seconds between loops of auditory stimuli series and 10 ms between beats. However, we also observed a high inter-variability, and decided to allow customization of this feature, so the inter and intra-trial interval would be in accordance to children's own pace and rhythm (see Lesson Learned 7).

7.1.4. Performance Feedback

We designed the game to be constantly evaluating if the child needs guidance to proceed in the task. If in the current task the child has provided two wrong solutions in a row and there is no manipulation of the tangible blocks for more than 30 seconds, the system reproduces auditory feedback to help the child to proceed. The game gives no direct indications, such as the exact number of missing blocks; instead, the feedback indicates if there are too few or too many blocks in the detection area. This generic feedback aims to help children to work out on their own to achieve the correct solution. To motivate children to proceed in the game, and keep them engaged, we provided diverse sounds that indicate that the child correctly solved the task. Each of these sounds is thematically related to the level that the child is completing at the moment. For example, if the child is mixing magical liquids, after solving correctly a task the system congratulates the child and plays a funny background sound of liquids and mixing.

7.1.5. Menu and volume settings

We provided the game with a menu composed of three buttons that allow the child to get back to the tutorial, restart or exit the game. Children could use the keyboard keys UP and DOWN to move from one item to the other, and ENTER to confirm their selection. When the child used UP and DOWN buttons, the corresponding audio item ("help", "restart" or "exit") was reproduced. The keyboard keys LEFT and RIGHT were used to decrease or increase the volume of the block sounds.

7.1.6. Audio Design

To ease the learning experience, we decided to use diegetic sounds originated by the game's universe (e.g., birds chirping, water running) and special effects to make the game experience richer and more engaging. We also included sounds with different pitches and timbres to facilitate discrimination between stimuli (see Lesson Learned 7), depicting known sounds as instruments, water drops, or door knocks, which also served to enhance the narrative, and provide a more engaging and immersive experience (Lesson Learned 9). For the design of task sounds, we used different auditory resources to ease sound perception, foster discrimination and facilitate counting following earcon - short synthetic tones usually employed in computer interfaces - design guidelines [55, 56] to increase auditory stimuli recognition and differentiation using timbre, rhythm and duration, intensity and spatial location. For the block sound and the task sound, we managed three parameters for the final prototype: tempo, pitch and timbre because they are the most efficient when it comes to discriminate[55, 56, 58]. Our approach is that each manipulative, from 1 to 5, has a unique or differentiable timbre, and an accent at the beginning. Number vocalisation is only presented in the tutorial to help children to identify the number while counting the number of beats. Later on there is no vocalisation of the number so that children are encouraged to pay attention to and count the number of beats.

Timbre. When we added the narrative to the computer task we were able to develop task sounds with different timbres related to the action required (e.g., steps, knocking the door, stirring the magic potion). We created a variety of timbres which changed depending on the different scenarios of the story to increase children's timbre recognition and differentiation (Lesson Learned 7).

Rhythm, duration and intensity. We were especially interested in designing stimuli that could be "heard at a glimpse" as a way to foster the simultaneous perception of a set in the auditory modality [4]. A series of sounds should be rhythmic to be easily counted, thus, our audio stimuli were composed by regularly distributed beats. Also, because it is more demanding to detect the number of items in a series when sets are up to 5-6 items [59] we decided to group larger numbers to facilitate counting; and task sounds were composed by series up to five beats. Beyond this number, quantities were expressed combining subsets of beats/ numbers. We added different sound intensities to increase discrimination of numbers. We used the following sound intensities for each number- considering S for strong, M for medium and L for lower volume: 1 - S, 2 - SL, 3 - SLL, 4-SLML, 5- SLLML.

Spatial location. The use of different audio spatial locations, enhanced by using headphones served to improve sound discrimination (Lesson Learned 7): block sounds were emitted to the right ear, and the audio corresponding to the task to the left.

8. iCETA Preliminary Evaluation at schools

To evaluate the system's potential to aid in mathematical learning we quantitatively assessed mathematics, working memory and haptic skills before and after children interacted with iCETA. We also observed how children interacted with iCETA during 15 sessions at their schools. We aimed to answer our RQ4: *Does the developed system and audiogame support maths training?*

8.1. Procedure: 3-week deployment at schools

Eleven children with visual impairments from school 1 and 2 participated in the evaluation. Some of them had already participated in the design sessions of iCETA (Table 1). We conducted individual pre-test assessments for approximately 30-60 minutes in 1 or 2 sessions depending if children needed more time or were tired. Then, each child played independently and autonomously with iCETA on their computers for 15 minutes for 15 sessions for 3 weeks at their schools. After, we conducted the same assessments as post-tests.

Numerical ability was assessed by combining tasks of TEMA-3 [60] and of Benton and Luria Battery Test [61]: (1) Count up task [60]; (2) comparison between two numbers [61]; (3) complete oral sequences of numbers [60]; (4) oral calculation [61]; (5) count tactile elements -one by one and grouping [61]; (6) count following the numerical verbal sentence [61]. We used the Haptic Test Battery [62] for children with visual impairments to assess their haptic perception, with two tests: the Dot Span Test and Object Span. Finally, short term memory was assessed with the task Digit Span Forward and working memory with the Digit Span Backward, both subtests of the WISC IV intelligence test [63].

8.2. Findings

The use of iCETA was successfully employed at both schools. Children were generally engaged and were capable of using the tangible blocks to solve the activities and progress in the game. The majority of children already had a basic understanding of numbers and number-line and were able to progress in the game and stay motivated even when they only knew to count up to 10.

8.2.1. Children were engaged and understood game dynamics

In general, children were engaged, liked and understood the game dynamics and tasks, except one participant that showed lack of general motivation. The youngest participant, 5 years old, was engaged and motivated in almost all the sessions and was one of the children who most quickly adapted to the game dynamics. On the other hand, other participants needed 3 sessions to totally understand the game dynamics. For two participants it was especially challenging to learn the game dynamics- both the discrete counting of the beats and the differentiation of task sound and blocks sound were quite challenging for them. Although they were interested in the game, mainly due to the shape and colours of the blocks and the narrative that seemed to catch their attention, the activities were too challenging for them. It was also challenging for another participant because she had difficulties in counting. However, her engagement with the audiogame made her willing to repeat activities, and consequently, to progress in the game.

One participant completed the game twice. She had moderate low-vision, good counting skills and she quickly understood the different values of the blocks and was very proficient with the use of the blocks. However, there were children that needed more time practising and that never finished the game. We think that to motivate and engage more children the game could have different levels of difficultly (e.g., easy, medium, hard). This feature would allow children to complete the game by choosing the appropriate level of difficulty.

8.2.2. Children improved at maths after training with iCETA

Children progressed in the game which required them to solve tasks progressively more difficult. We calculated if there was a gain between the pre-test and post-tests for each ability assessed -numerical, haptic and short and working memory. Our analysis indicates that after playing Logarin, children significantly improved their numerical ability ($t(8)$=-2.59, p = 0.03) whereas we did not observed any significant improvement for short and working memory and haptic abilities. We followed up the numerical ability analysis with paired t-test and we found that children increased their oral counting ability ($t(8)$=-2.25, p = 0.05). However, as we did not have a control group we cannot be certain about the implications of these results. Our preliminary results are promising and further studies should be conducted to shed light on the possible outcomes.

8.2.3. Children varied in their strategies with the blocks to solve the tasks

We observed different strategies to solve additive compositions. Some children regularly used the blocks of 1 to compose any number. This fact could be related to the lack of understanding of the blocks as representations of numbers, to the maths level of the child or just as a matter of preference of using those specific blocks. In fact, one participant constantly used blocks of 1, but when asked to use higher value blocks he could easily solve the activities as well. Another participant, on the other hand, when asked to use greater blocks had problems to find a correct solution. For this participant, it was difficult to understand that blocks represented different numbers. The participant that solved the game twice developed her own strategy to solve the tasks; she joined a set of blocks together at the table and count with her fingers the units of the blocks before introducing the blocks to the detection area.

8.2.4. Personalization by children

We designed the inter and intra-trial settings to be adjusted by the researchers or educators. The configuration panel was fully visual, without auditory clues to guide children to its location and to change the inter and intra-trial durations the user needed to use screen-based buttons. However, after interacting with children in this evaluation we consider that it may be valuable to enable children to access this menu and to adapt the timings at their will. To do so, we will consider the use of keyboard keys to increase or decrease the inter and intra-trial duration.

8.2.5. Detection of the blocks

Due to the different light conditions in schools, TopCodes were not always easily detected and the researchers had to control the artificial light at the room or to close the room's curtains, for instance. We could overcome this challenge in the future by providing a possibility of an initial calibration of the TopCodes using iCETA. In this way, the system could record the actual light conditions of the room and adjust the detection of TopCodes to the current setting. We also found that the detection of block of value 1 was more difficult. This is mainly due to the fact that this block has only one TopCode decreasing the detection probability compared to the other blocks that have more than one TopCode. We could add another marker to the block to increase it's detection rate.

9. Discussion

In this paper, we describe the process of a PD to develop an accessible and low cost TUI to support the training of basic maths skills. In this study, we contribute with (1) iCETA - developed in collaboration with children, considering their context, needs and developmental skills; (2) description of the Lessons Learned during the PD with children at each phase of the system development (summarized in Tables 2 and 3); and (3) a preliminary evaluation of iCETA during 15 sessions with children in 2 special schools.

We developed iCETA to provide an accessible, playful and rich multi-sensorial environment for children with visual impairments to learn maths. iCETA allows children to perform sensory-motor actions with tangibles to learn basic operations with numbers. Besides, to develop active touch counting strategies [2], tangibles had a clear structure emphasizing its mathematical concept. At the same time, iCETA includes a series of auditory beats used to represent numbers which can be counted. This sensory-based experience also strengthens the conceptual abstraction required to understand arithmetic operations. Importantly, the multisensory aspects of iCETA afforded a greater inclusion of children with visual impairments and cognitive or developmental impairments.

9.1. PD experiences in Special Needs School-Communities

The team that actively participated during the PD were composed of one psychologist, one designer and one student of psychology. We already had a background on cognitive development, design and education, but we needed to be integrated and embedded in children's activities and to be part of their daily lives at school. Stakeholders valued the fact that researchers had already background knowledge on children's development and education which strengthened their welcoming and the relationship between both parts.

Our context of children was very diverse; they had different ages, types of visual impairments, types of mental and motor development, autism or Attention Deficit Hyperactivity Disorder. We aimed to develop a TUI that could be used by children with different abilities and different maths levels, creating a useful tool for those school-communities. We used a hybrid approach [64], where researchers were embedded in children's context (i.e. their schools) which roles varied from friend or observer, to the role of leader or supervisor [64] balancing between degrees of agency of the researchers and those of the children depending on the situation, child or activity.

The embedding of researchers at schools had ethical and political consequences [65] as it affected their communities, such as staff, directors, teachers and children (not only the children we worked with). We wanted to be part of the community, to be their friends, so we could have a positive effect in those communities; we accompanied children at a variety of activities, such as at lunch, classes, Christmas activities, playground, etc. It is important to note that this research took place during one year and a strong relationship and bonding between children and the researchers were built. This positive relationship was further extended to the school communities who were also enthusiastic with our regular visits. Before each workshop we had informal conversations, we played informally and some children liked to play musical instruments for us. We brought children from their class to the library or to the art's room where the workshops were conducted. In the way, we had the opportunity to talk to each child individually, engaging in daily conversations and with time they got to know us better; which were our motivations, preferences, stories and families. We all needed that space to construct a fruitful relationship and to get to know each other better. We learned their personal fears and dislikes which we took into account for the design of our workshops and audiogame - e.g., one autistic child had fear of guitar sounds so we took off that sound for the next iterations and prototypes. We also brought prizes or rewards each week for the children. Children could ask specifically simple objects they like us to bring (e.g., bottle caps of different colours was repeatedly requested by a blind boy) or that we took especially for them (e.g., pencils of different colours, party objects, 3D Logarin character, etc.). As researchers, we had the aim to increase children's agency and self-esteem and to empower them by actively listening to their voices, needs and preferences - the majority of these children were part of families with low socio-economic status and they are often marginalized at their homes and in their communities (besides school) so we wanted to be a positive influence in their lives.

9.2. Design Considerations to Develop an Inclusive TUI

We reflected on our lessons learned (Tables 2 and 3) and derived some design considerations that researchers, developers and designers may consider when developing an accessible TUI for children with visual impairments, not specifically for maths.

An important consideration when designing tangibles is that tactile perception is serial and not parallel as vision [66]. This implies that if tactile cues (as the ones we used: hollow and raised markers) are located too near this may impair finger's sensory discrimination which in turn difficult the integration of the components into a coherent whole. We recommend being **parsimonious with the use of tactile cues** as it can be confusing to perceive and identify cues if they are too many and too near. When we changed the form of the tangibles, divisions were more naturally perceived, almost as if the tangibles were composed of circles, without overloading children with tactile cues.

Another consideration when designing tangibles is to **include braille** as it has a great implication in the life of people with visual impairments as it is a precursor of academic and professional success [67]. In line with what was reinforced by educators, we also foresee the opportunity for children to learn braille by embedding it in tangibles, as it could be another vehicle to learn braille. It would be relevant and beneficial to label tangibles with braille so children could learn braille while identifying the tangible. For the recent studies that plan to incorporate braille into their systems [19, 51] we find it important to stress that, when adding braille labels to tangibles, developers and designers must be aware of **braille's right dimensions** considering braille guidelines and the need to also create **affordances to correctly orient braille** to an effective braille reading.

Additionally, we recommend the use of a **storage box** where the tangibles could be stored, ordered and labelled in braille. The opportunity to have tangibles stored and categorized would free-up cognitive resources that otherwise would be needed to memorise the different tangibles and their locations.

To create an accessible TUI for children with visual impairments, the auditory channel is frequently the most stimulated followed by the haptic channel. Due to individual differences and preferences, we recommend some level of **auditory customization** by the users. Also, because a great part of the information would be conveyed by auditory stimuli, it would be important to use **earcons guidelines** [55, 56] to facilitate auditory discrimination. For instance, using binaural sounds allow children to sense auditory information that arrives at different ears which help in distinguishing auditory stimuli.

10. Conclusion and Future Work

We aimed to develop a low-cost TUI to be used in special schools for children with visual impairments. We followed a PD approach with stakeholders to develop a tangible system intended to provide better scaffolding for understanding the abstract concept of numbers within an audiogame. Working with children and educators allowed us to detect different design opportunities related to how to enhance number detection through auditory and haptic stimuli that can also be used to design new devices for people with visual impairments.

We attempted to create a TUI that could be used for almost all the children in both schools integrating their heterogeneity and comorbidity (besides their visual impairment some children had cognitive or developmental difficulties). However, further research must be carried out to study and create more activities and tools suited to different levels of cognitive and motor development. Our preliminary evaluation of iCETA evidence that this line is worth pursuing but a depth study with a control group would be necessary to assess the efficacy of iCETA and the audiomath game Logarin.

One of our future goals is also to expand the number of math concepts in Logarín and to explore the use of intelligent blocks with built-in electronics to provide more diverse feedback (speech, sound, vibration and force), supporting novel ways of interaction and learning in the context of children with visual impairments [68]. We are also currently exploring different accessible TUI environments to learn computational thinking and engage children in spatial activities with a robot [21, 23, 22, 24, 69]. Results revealed that accessible TUIs are valuable and affordable to use at schools for a positive impact on children's learning.

Acknowledgments

We would like to thank all schools, staff, educators, and children for their enthusiastic participation in the reported studies. This work was supported by Agencia Nacional de Investigación e Innovación and Fundación Ceibal (FSED2-2016-1-131112), by Espacio Interdisciplinario, UDELAR, Uruguay and by Fundação para a Ciência e a Tecnologia (FCT), I.P., through project mIDR (AAC 02/SAICT/-2017, project 30347, cofunded by COMPETE/FEDER/FNR), and the LASIGE Research Unit ref. UIDB/00408/2020 and ref. UIDP/00408/2020, Portugal.

References

[1] A. Manches, C. O'Malley, The effects of physical manipulatives on children's numerical strategies, Cognition and Instruction 34 (1) (2016) 27–50. doi:10.1080/07370008.2015.1124882.

[2] A. Ahlberg, E. Csocsán, Grasping numerosity among blind children., Department of Education and Educational Research, Goeteborg University., 1994.

[3] S. P. Sicilian, Development of counting strategies in congenitally blind children, Journal of Visual Impairment and Blindness 82 (8) (1988) 331–335.

[4] J. Leuders, Tactile and acoustic teaching material in inclusive mathematics classrooms, British Journal of Visual Impairment 34 (1) (2015) 42–53. doi:10.1177/0264619615610160.

[5] A. Ahlberg, E. Csocsán, How children who are blind experience numbers, Journal of Visual Impairment & Blindness 93 (9) (1999) 549–560. doi:10.1177/0145482X9909300902.

[6] V. Crollen, R. Mahe, O. Collignon, X. Seron, The role of vision in the development of finger–number interactions: Finger-counting and finger-montring in blind children, Journal of experimental child psychology 109 (4) (2011) 525–539.

[7] L. B. Resnick, A developmental theory of number understanding. URL http://files.eric.ed.gov/fulltext/ED251328.pdf

[8] P. Hatlen, The core curriculum for blind and visually impaired students, including those with additional disabilities., 1996.

[9] M. A. Mastropieri, T. E. Scruggs, The inclusive classroom: Strategies for effective differentiated instruction, Pearson, 2017.

[10] G. Lakoff, R. Núñez, Where mathematics comes from, Vol. 6, New York: Basic Books, 2000.

[11] S. P. Miller, P. J. Hudson, Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge, Learning Disabilities Research & Practice 22 (1) (2007) 47–57. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-5826.2007.00230.x, doi:https://doi.org/10.1111/j.1540-5826.2007.00230.x.
URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-5826.2007.00230.x

[12] S. Amato, S. Hong, L. P. Rosenblum, The abacus: Instruction by teachers of students with visual impairments, Journal of Visual Impairment & Blindness 107 (4) (2013) 262–272. arXiv:https://doi.org/10.1177/0145482X1310700403, doi:10.1177/0145482X1310700403.
URL https://doi.org/10.1177/0145482X1310700403

[13] S. M. Millaway, Abacus basic competency: A counting method, American Printing House for the Blind, USA, 2001.

[14] M. K. Stein, J. W. Bovalino, Reflections on practice: Manipulatives: One piece of the puzzle, Mathematics Teaching in the Middle School 6 (6) (2001) 356–359.

[15] S. Osterhaus, Teaching math to visually impaired students (1996).
URL http://www.tsbvi.edu/math

[16] R. H. Craig, Learning the Nemeth Braille Code: A Manual for Teachers, Brigham Young University Press, 1979.

[17] R. Jafri, Electronic braille blocks: A tangible interface-based application for teaching braille letter recognition to very young blind children, in: K. Miesenberger, D. Fels, D. Archambault, P. Peňáz, W. Zagler (Eds.), Computers Helping People with Special Needs, Springer International Publishing, Cham, 2014, pp. 551–558.

[18] V. Gadiraju, A. Muehlbradt, S. K. Kane, Brailleblocks: Computational braille toys for collaborative learning, in: Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems, CHI '20, Association for Computing Machinery, New York, NY, USA, 2020, p. 1–12. doi:10.1145/3313831.3376295.

[19] V. Koushik, D. Guinness, S. K. Kane, Storyblocks: A tangible programming game to create accessible audio stories, in: Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems, CHI '19, ACM, New York, NY, USA, 2019, pp. 492:1–492:12. doi:10.1145/3290605.3300722.

[20] A. Thieme, C. Morrison, N. Villar, M. Grayson, S. Lindley, Enabling collaboration in learning computer programing inclusive of children with vision impairments, in: Proceedings of the 2017 Conference on Designing Interactive Systems, DIS '17, ACM, New York, NY, USA, 2017, pp. 739–752. doi:10.1145/3064663.3064689.

[21] A. C. Pires, F. Rocha, A. J. de Barros Neto, H. Simão, H. Nicolau, T. Guerreiro, Exploring accessible programming with educators and visually impaired children, IDC '20, Association for Computing Machinery, New York, NY, USA, 2020, p. 148–160. doi:10.1145/3392063.3394437.

[22] F. Rocha, A. C. Pires, I. Neto, H. Nicolau, T. Guerreiro, Accembly at home: Accessible spatial programming for children with visual impairments and their families, IDC '21, Association for Computing Machinery, New York, NY, USA, 2021. doi:10.1145/3459990.3460699.

[23] L. Abreu, A. C. Pires, T. Guerreiro, Tactopi: A playful approach to promote computational thinking for visually impaired children, in: The 22nd International ACM SIGACCESS Conference on Computers and Accessibility, ASSETS '20, Association for Computing Machinery, New York, NY, USA, 2020. doi:10.1145/3373625.3418003.

[24] G. Cardoso, A. C. Pires, L. V. Abreu, F. Rocha, T. Guerreiro, Legoworld: Repurposing commodity tools & technologies to create an accessible and customizable programming environment, in: Extended Abstracts of the 2021 CHI Conference on Human Factors in Computing Systems, CHI EA '21, Association for Computing Machinery, New York, NY, USA, 2021. doi:10.1145/3411763.3451710.
URL https://doi.org/10.1145/3411763.3451710

[25] R. Jafri, A. M. Aljuhani, S. A. Ali, A tangible interface-based application for teaching tactual shape perception and spatial awareness sub-concepts to visually impaired children, Procedia Manufacturing 3 (2015) 5562–5569. doi:10.1016/j.promfg.2015.07.734.

[26] R. Jafri, A. M. Aljuhani, S. A. Ali, A tangible user interface-based application utilizing 3d-printed manipulatives for teaching tactual shape perception and spatial awareness sub-concepts to visually impaired children, International Journal of Child-Computer Interaction 11 (2017) 3 – 11, designing with and for Children with Special Needs. doi:https://doi.org/10.1016/j.ijcci.2016.12.001.

[27] L. M. Rühmann, N. Otero, I. Oakley, A tangible tool for visual impaired users to learn geometry, in: Proceedings of the TEI '16: Tenth International Conference on Tangible, Embedded, and Embodied Interaction, TEI '16, Association for Computing Machinery, New York, NY, USA, 2016, p. 577–583. doi:10.1145/2839462.2856536.

[28] M. S. Manshad, E. Pontelli, S. J. Manshad, Trackable interactive multimodal manipulatives: towards a tangible user environment for the blind, in: International Conference on Computers for Handicapped Persons, Springer, 2012, pp. 664–671.

[29] D. McGookin, E. Robertson, S. Brewster, Clutching at straws: Using tangible interaction to provide non-visual access to graphs, in: Proceedings of the SIGCHI Conference on Human Factors in Computing Systems, CHI

'10, Association for Computing Machinery, New York, NY, USA, 2010, p. 1715–1724. doi:10.1145/1753326.1753583.

[30] M. S. Manshad, E. Pontelli, S. J. Manshad, Micoo (multimodal interactive cubes for object orientation), The proceedings of the 13th international ACM SIGACCESS conference on Computers and accessibility - ASSETS 11doi:10.1145/2049536.2049597.

[31] M. S. Manshad, E. Pontelli, S. J. Manshad, Exploring tangible collaborative distance learning environments for the blind and visually impaired, in: CHI '13 Extended Abstracts on Human Factors in Computing Systems, CHI EA '13, Association for Computing Machinery, New York, NY, USA, 2013, p. 55–60. doi:10.1145/2468356.2468367.

[32] M. Ávila Soto, E. Valderrama-Bahamóndez, A. Schmidt, Tanmath: A tangible math application to support children with visual impairment to learn basic arithmetic, in: Proceedings of the 10th International Conference on PErvasive Technologies Related to Assistive Environments, PETRA '17, Association for Computing Machinery, New York, NY, USA, 2017, p. 244–245. doi:10.1145/3056540.3064964.

[33] C. Magnusson, H. Caltenco, S. Finocchietti, G. Cappagli, G. Wilson, M. Gori, What do you like? early design explorations of sound and haptic preferences, in: Proceedings of the 17th International Conference on Human-Computer Interaction with Mobile Devices and Services Adjunct, MobileHCI '15, Association for Computing Machinery, New York, NY, USA, 2015, p. 766–773. doi:10.1145/2786567.2793699.

[34] O. Metatla, C. Cullen, "bursting the assistance bubble": Designing inclusive technology with children with mixed visual abilities, in: Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems, CHI '18, Association for Computing Machinery, New York, NY, USA, 2018. doi:10.1145/3173574.3173920.

[35] C. Morrison, N. Villar, A. Thieme, Z. Ashktorab, E. Taysom, O. Salandin, D. Cletheroe, G. Saul, A. F. Blackwell, D. Edge, M. Grayson, H. Zhang, Torino: A tangible programming language inclusive of children with visual disabilities, Human–Computer Interaction 35 (3) (2020) 191–239. doi:10.1080/07370024.2018.1512413.

[36] O. Metatla, A. Oldfield, T. Ahmed, A. Vafeas, S. Miglani, Voice user interfaces in schools: Co-designing for inclusion with visually-impaired and sighted pupils, in: Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems, CHI '19, Association for Computing Machinery, New York, NY, USA, 2019, p. 1–15. doi:10.1145/3290605.3300608.

[37] O. Metatla, S. Bardot, C. Cullen, M. Serrano, C. Jouffrais, Robots for inclusive play: Co-designing an educational game with visually impaired and sighted children, in: Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems, CHI '20, Association for Computing Machinery, New York, NY, USA, 2020, p. 1–13. doi:10.1145/3313831.3376270.

[38] J. Waddington, C. Linehan, K. Gerling, K. Hicks, T. L. Hodgson, Participatory design of therapeutic video games for young people with neurological vision impairment, in: Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems, CHI '15, Association for Computing Machinery, New York, NY, USA, 2015, p. 3533–3542. doi:10.1145/2702123.2702261.

[39] G. Regal, D. Sellitsch, S. Kriglstein, S. Kollienz, M. Tscheligi, Be active! participatory design of accessible movement-based games, in: Proceedings of the Fourteenth International Conference on Tangible, Embedded, and Embodied Interaction, TEI '20, Association for Computing Machinery, New York, NY, USA, 2020, p. 179–192. doi:10.1145/3374920.3374953.

[40] O. Metatla, C. Cullen, "Bursting the Assistance Bubble": Designing Inclusive Technology with Children with Mixed Visual Abilities, Association

for Computing Machinery, New York, NY, USA, 2018, p. 1–14. URL https://doi.org/10.1145/3173574.3173920

[41] E. Brulé, G. Bailly, Taking into account sensory knowledge: The case of geo-techologies for children with visual impairments, in: Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems, CHI '18, Association for Computing Machinery, New York, NY, USA, 2018. doi:10.1145/3173574.3173810.

[42] J. McElligott, L. van Leeuwen, Designing sound tools and toys for blind and visually impaired children, in: Proceedings of the 2004 Conference on Interaction Design and Children: Building a Community, IDC '04, Association for Computing Machinery, New York, NY, USA, 2004, p. 65–72. doi:10.1145/1017833.1017842.

[43] C. Magnusson, H. Caltenco, S. Finocchietti, G. Cappagli, G. Wilson, M. Gori, What do you like? early design explorations of sound and haptic preferences, in: Proceedings of the 17th International Conference on Human-Computer Interaction with Mobile Devices and Services Adjunct, MobileHCI '15, Association for Computing Machinery, New York, NY, USA, 2015, p. 766–773. doi:10.1145/2786567.2793699.

[44] E. Brule, G. Bailly, A. Brock, F. Valentin, G. Denis, C. Jouffrais, Mapsense: Multi-sensory interactive maps for children living with visual impairments, in: Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems, CHI '16, Association for Computing Machinery, New York, NY, USA, 2016, p. 445–457. doi:10.1145/2858036.2858375.

[45] V. Nesset, A. Large, Children in the information technology design process: A review of theories and their applications, Library & Information Science Research 26 (2) (2004) 140–161.

[46] A. Druin, The role of children in the design of new technology, Behaviour and information technology 21 (1) (2002) 1–25. doi:10.1080/01449290110108659.

[47] W. Xu, K. Zammit, Applying thematic analysis to education: A hybrid approach to interpreting data in practitioner research, Vol. 19, 2020, p. 1609406920918810. arXiv:https://doi.org/10.1177/1609406920918810, doi:10.1177/1609406920918810.
URL https://doi.org/10.1177/1609406920918810

[48] S. Marichal, A. Rosales, F. G. Perilli, A. C. Pires, E. Bakala, G. Sansone, J. Blat, Ceta: Designing mixed-reality tangible interaction to enhance mathematical learning, Proceedings of the 19th International Conference on Human-Computer Interaction with Mobile Devices and Services - MobileHCI 17doi:10.1145/3098279.3098536.

[49] A. C. Pires, F. González Perilli, E. Bakała, B. Fleisher, G. Sansone, S. Marichal, Building blocks of mathematical learning: Virtual and tangible manipulatives lead to different strategies in number composition, Frontiers in Education 4 (2019) 81. doi:10.3389/feduc.2019.00081.

[50] M. D. Lozano, V. M. R. Penichet, B. Leporini, A. Fernando, Tangible user interfaces to ease the learning process of visually-impaired children, in: Proceedings of the 32nd International BCS Human Computer Interaction Conference, HCI '18, BCS Learning & Development Ltd., Swindon, GBR, 2018. doi:10.14236/ewic/HCI2018.87.
URL https://doi.org/10.14236/ewic/HCI2018.87

[51] F. da Rocha Tomé Filho, P. Mirza-Babaei, B. Kapralos, G. Moreira Mendonça Junior, Let's play together: Adaptation guidelines of board games for players with visual impairment, in: Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems, CHI '19, Association for Computing Machinery, New York, NY, USA, 2019, p. 1–15. doi:10.1145/3290605.3300861.
URL https://doi.org/10.1145/3290605.3300861

[52] A. Sabuncuoglu, Tangible music programming blocks for visually impaired children, in: Proceedings of the Fourteenth International Conference on

Tangible, Embedded, and Embodied Interaction, TEI '20, Association for Computing Machinery, New York, NY, USA, 2020, p. 423–429. doi:10.1145/3374920.3374939.
URL https://doi.org/10.1145/3374920.3374939

[53] M. Horn, Topcode: Tangible object placement codes (2012).
URL http://users.eecs.northwestern.edu/~mhorn/topcodes/

[54] A. B. Najjar, A. Alhussayen, R. Jafri, Usability engineering of a tangible user interface application for visually impaired children, Human-centric Computing and Information Sciences 11 (14) (2021) 891–921. doi:https://doi.org/10.22967/HCIS.2021.11.014.

[55] S. A. Brewster, P. C. Wright, A. D. Edwards, A detailed investigation into the effectiveness of earcons, in: Santa Fe Institute Studies in The Sciences of Complexity, Vol. 18, Addison-Wesley Publishing CO, 1994, pp. 471–471.

[56] S. A. Brewster, P. C. Wright, A. D. Edwards, Experimentally derived guidelines for the creation of earcons, in: Adjunct Proceedings of HCI, Vol. 95, 1995, pp. 155–159.

[57] J.-A. LeFevre, J. Morris, More on the relation between division and multiplication in simple arithmetic: Evidence for mediation of division solutions via multiplication, Memory & cognition 27 (5) (1999) 803–812.

[58] S. A. Brewster, P. C. Wright, A. D. Edwards, An evaluation of earcons for use in auditory human-computer interfaces, in: Proceedings of the INTERACT'93 and CHI'93 conference on Human factors in computing systems, 1993, pp. 222–227.

[59] B. H. Repp, Perceiving the numerosity of rapidly occurring auditory events in metrical and nonmetrical contexts, Perception & Psychophysics 69 (4) (2007) 529–543.

[60] H. P. Ginsburg, A. J. Baroody, Tema-3: Test of early mathematics ability–third edition.

[61] L. Benton, Chadwick, Benton-luria. evaluación del conocimiento matemático.

[62] S. Ballesteros, D. Bardisa, S. Millar, J. M. Reales, The haptic test battery: A new instrument to test tactual abilities in blind and visually impaired and sighted children, British Journal of Visual Impairment 23 (1) (2005) 11–24.

[63] D. Wechsler, Wechsler preschool and primary scale of intelligence—fourth edition, The Psychological Corporation San Antonio, TX.

[64] W. Barendregt, P. Börjesson, E. Eriksson, O. Torgersson, T. Bekker, H. M. Skovbjerg, Modelling the roles of designers and teaching staff when doing participatory design with children in special education, in: Proceedings of the 15th Participatory Design Conference: Full Papers - Volume 1, PDC '18, Association for Computing Machinery, New York, NY, USA, 2018. doi:10.1145/3210586.3210589.

[65] O. S. Iversen, R. C. Smith, C. Dindler, Child as protagonist: Expanding the role of children in participatory design, in: Proceedings of the 2017 Conference on Interaction Design and Children, IDC '17, Association for Computing Machinery, New York, NY, USA, 2017, p. 27–37. doi:10.1145/3078072.3079725.

[66] M. A. Heller, E. Gentaz, Psychology of touch and blindness, Psychology press, 2013.

[67] A. M. Silverman, E. C. Bell, The association between braille reading history and well-being for blind adults., Journal of Blindness Innovation & Research 8 (1).

[68] S. Marichal, A. Rosales, G. Sansone, A. C. Pires, E. Bakala, F. G. Perilli, B. Fleischer, J. Blat, Letsmath, in: Proceedings of the 20th International Conference on Human-Computer Interaction with Mobile Devices and Ser-

vices Adjunct, MobileHCI '18, Association for Computing Machinery, New York, NY, USA, 2018, p. 313–320. doi:10.1145/3236112.3236157.

[69] F. Rocha, G. Guimarães, D. Gonçalves, A. C. Pires, L. Abreu, , T. Guerreiro, Fostering collaboration with asymmetric roles in accessible programming environments for children with mixed-visual-abilities, in: The 23nd International ACM SIGACCESS Conference on Computers and Accessibility, ASSETS '21, Association for Computing Machinery, New York, NY, USA, 2021. doi:10.1145/3441852.3476553.