LOCAL NON-BOSSINESS

EDUARDO DUQUE-ROSAS, JUAN S. PEREYRA, AND JUAN PABLO TORRES-MARTÍNEZ

ABSTRACT. The student-optimal stable mechanism (DA), the most popular mechanism in school choice, is the only one that is both stable and strategy-proof. However, when DA is implemented, a student can change the schools of others without changing her own. We show that this drawback is limited: a student cannot change her classmates while remaining in the same school. We refer to this new property as *local non-bossiness* and use it to provide a new characterization of DA that does not rely on stability. Furthermore, we show that local non-bossiness plays a crucial role in providing incentives to be truthful when students have preferences over their colleagues. As long as students first consider the school to which they are assigned and then their classmates, DA induces the only stable and strategy-proof mechanism. There is limited room to expand this preference domain without compromising the existence of a stable and strategy-proof mechanism.

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E. Duque-Rosas, London School of Economics and Political Science (e.g.duque@lse.ac.uk).

J. S. Pereyra, Universidad de Montevideo, Uruguay (jspereyra@um.edu.uy).

J. P. Torres-Martínez, Department of Economics, Faculty of Economics and Business, University of Chile (juan.torres@fen.uchile.cl).

1. INTRODUCTION

In the last two decades, an increasing number of centralized school admission systems have been implemented worldwide.[1] In this context, the *student-optimal stable mechanism* (DA) of Gale and Shapley (1962) has become one of the most popular mechanisms for distributing school seats. These developments are rooted in the theoretical results laid out in the seminal works of Roth (1985), Roth and Sotomayor (1989), Balinski and Sönmez (1999), Abdulkadiroğlu and Sönmez (2003), and Pathak and Sönmez (2013).[2]

The success of DA might be explained by its properties. When students have strict preferences for schools, it is well-known that DA is the only stable and strategy-proof mechanism (Dubins and Freedman, 1981; Roth, 1982; Alcalde and Barberà, 1994). Stability ensures that school seats are not wasted and that no one would prefer the seat of someone with lower priority for it (i.e., there is no "justified envy"). Strategy-proofness guarantees that no student has incentives to misreport her preferences.

However, when DA is implemented, a change in a student's preference can modify the assignment of others without changing her own (Roth, 1982). In technical terms, DA is *bossy*, and this is important for at least two reasons. First, it is related to the Pareto inefficiency of DA and to the fact that a coalition of students could improve their assignments by jointly misrepresenting their preferences (Papái, 2000; Ergin, 2002). Second, it has significant consequences for incentives when students care not only about the school to which they are assigned but also about the assignment of others. Indeed, it is the bossiness of DA that compromises the existence of a stable and strategy-proof mechanism when there are externalities but each student prioritizes her own school (Duque and Torres-Martínez, 2023).

In this paper, we introduce a new incentive property satisfied by DA that limits its bossiness. The new property, called *local non-bossiness*, says that a

[1]The website www.ccas-project.org provides information about school choice systems in different countries.

[2]We refer to the surveys of Abdulkadiroğlu (2013), Kojima (2017), and Pathak (2017) for a discussion of recent developments in school choice and their applications.

student cannot change her classmates without changing the school to which she is assigned. Thus, when a locally non-bossy mechanism is used, a student's capacity to modify others' assignment without changing her school is limited to those who are not assigned to the same school. This includes the case where the student is not assigned: she cannot modify the set of unassigned students while remaining without school. Local non-bossiness is therefore non-trivial even in a one-to-one matching problem. We use local non-bossiness, along with a set of axioms that do not include stability, to characterize the DA mechanism. We also study a model with externalities and show that local non-bossiness ensures the existence of a stable and strategy-proof mechanism when students care first about the school to which they are assigned, and then about their classmates.

Our analysis begins by showing that DA is locally non-bossy (Theorem 1) and that this property is independent of both stability or strategy-proofness. Furthermore, any locally non-bossy and strategy-proof mechanism is *locally group strategy-proof*, in the sense that no coalition of *classmates* can manipulate it to improve the situation for at least one of its members without hurting any other member. In particular, DA is the only stable and locally group strategy-proof mechanism (Corollary 1). We also study the relationships between local non-bossiness, local group strategy-proofness, and other incentive properties (the Figure 1 summarizes our results).

Secondly, we use local non-bossiness to provide a new characterization of DA that does not involve priorities. We fix a set of schools and capacities, and consider a mechanism as a function that assigns a matching to each set of students and their preferences. We show that a mechanism satisfies individual rationality, weak non-wastefulness, population-monotonicity, strategy-proofness, weak WrARP, and weak local non-bossiness if and only if it is the DA mechanism for some profile of priorities (Theorem 2). Under individual rationality no student is assigned to a school that she considers unacceptable; weak non-wastefulness ensures that unassigned students never prefer a school that did not fill its places; and population-monotonicity guarantees that no one gets worse off when the set of students shrinks. Along with strategy-proofness, these three axioms characterize DA when schools have only one seat available (Ehlers and Klaus, 2016).[3] Weak WrARP is a necessary condition to construct for each school a choice function consistent with a priority order (Chambers and Yenmez, 2018). Weak local non-bossiness restricts local non-bossiness to schools, allowing a student without school to modify the set of unassigned students without getting a seat somewhere. Weak WrARP and weak local non-bossiness make it possible to manage schools with more than one seat available, and they are trivially satisfied when each school has only one seat available. Strengthening strategy-proofness to group strategy-proofness, our axiomatization allows us to characterize DA under acyclic priority profiles (Corollary 2).

Thirdly, we introduce externalities in our model by allowing students to have preferences over the set of matchings. In particular, we analyze the implications of the local non-bossiness of DA in this context. It is well known that many of the results in the literature break down in the presence of externalities, for example, the existence of stable matching. One way to recover this result is by restricting preferences to be *school-lexicographic*. That is, by assuming that each student is primarily concerned with her assigned school, and when assigned to the same school in two different matchings, there is no restriction on how to compare them (Sasaki and Toda, 1996; Dutta and Massó, 1997; Fonseca-Mairena and Triossi, 2023).

However, even on this restricted domain, a stable and strategy-proof mechanism may not exist (Duque and Torres-Martínez, 2023). We further restrict the preference domain to the family of *school-lexicographic preferences over colleagues*, assuming that students care first about the school and then only about their classmates. That is, each student is indifferent among all the matchings in which she is assigned to the same school with the same classmates. There are no restrictions on the order in which she ranks two matchings that assign her to the same school but with different classmates. In this context, we demonstrate that applying DA to the school rankings underlying school-lexicographic preferences over colleagues induces a stable

[3]We thank Keisuke Bando for pointing out the relation between local non-bossiness and the characterization of Ehlers and Klaus (2016) for the unit capacity case.

and strategy-proof mechanism (Theorem 3). Intuitively, a student may want to misreport her preferences to either change her school or maintain it and change her classmates. The first reason for misreporting preferences is already present in classical school choice problems, and avoiding it relates to ensuring strategy-proofness. The second one emerges in the presence of school-lexicographic preferences, and avoiding it relates to guaranteeing local non-bossiness. Therefore, the local non-bossiness and strategy-proofness of DA are key to ensuring the existence of a stable and strategy-proof mechanism in this context.

Consequently, the incompatibility between stability and strategy-proofness in contexts where students prioritize their own school is not caused by the existence of preferences over the assignment of others *per se*, but by the fact that these preferences extend beyond their classmates. Evidently, it is reasonable to maintain the dependence of preferences on classmates because it is a well-documented empirical phenomenon in school choice (Rothstein, 2006; Abdulkadiroğlu et al., 2020; Allende, 2021; Che et al., 2022; Beuermann et al., 2023; Cox et al., 2023).

Our results have practical implications for admission systems based on the DA mechanism. They show that DA still performs well when students care about the assignments of others, as long as they first consider their assigned school and then their classmates. More precisely, when DA is implemented and students are only required to report strict rankings of schools, it is a weakly dominant strategy for each student with school-lexicographic preferences over colleagues to report the school ranking induced by her true preferences. Furthermore, despite the variety of mechanisms that could be defined using information on preferences for schools and classmates, DA induces the only stable and strategy-proof mechanism in this domain (Corollary 3). It is important to note that there is limited room to relax the assumption of school-lexicographic preferences over colleagues. Indeed, it is enough for the preferences of just one student to be school-lexicographic but not school-lexicographic over colleagues to prevent the existence of a stable and strategy-proof mechanism (see Section 5).

Related literature. We contribute to three strands of the literature: the analysis of the bossiness of DA, the axiomatization of DA without appealing to stability, and the study of school choice problems with externalities.

The concept of non-bossiness was first introduced by Satterthwaite and Sonnenschein (1981) and has been studied extensively in the context of the assignment of indivisible goods.[4] Non-bossiness plays an essential role in avoiding coalitional manipulability, as group strategy-proofness is equivalent to strategy-proofness and non-bossiness when preferences are strict (Papái, 2000). Moreover, for a given pair of schools' priorities and capacities, Ergin (2002) shows that DA is non-bossy if and only if it is Pareto efficient. Kojima (2010) demonstrates the incompatibility between non-bossiness and stability in college admission problems, a scenario where both sides of the market may act strategically. However, when only one side of the market reports preferences, as in school choice problems, Afacan and Dur (2017) show that the school-optimal stable mechanism is non-bossy for the students. We contribute to this strand of the literature by highlighting that the bossiness of DA is limited: when it is implemented, no student can change her classmates without also changing her assigned school. We also show that this property implies that no group of students assigned to the same school can manipulate DA to either improve the assignment of at least one of them without hurting any other member (locally group strategy-proofness), or maintain the school and change the other classmates (locally group non-bossiness).

The DA mechanism has been axiomatized in several ways. It is the only stable mechanism that is weakly Pareto efficient (Roth, 1982), and the only stable and strategy-proof mechanism (Alcalde and Barberà, 1994).[5] Balinski and Sönmez (1999) and Kojima and Manea (2010) provide alternative axiomatizations without appealing to strategy-proofness, while Kojima and Manea (2010), Morrill (2013), and Ehlers and Klaus (2014, 2016) characterize DA without appealing to stability. In particular, assuming that each school has

[4]We refer to the critical survey by Thomson (2016) for a detailed discussion of the multiple interpretations of non-bossiness and its implications.

[5]When students always report all schools as admissible, DA is not necessarily the only stable and strategy-proof mechanism (Sirguiado and Torres-Martínez, 2024).

only one seat available, Ehlers and Klaus (2016) show that a mechanism satisfies individual rationality, weak non-wastefulness, population-monotonicity, and strategy-proofness if and only if it is the DA for some priority profile. However, they show that this characterization does not hold in a general capacity model. By introducing two new axioms, weak WrARP and weak local non-bossiness, we extend Ehlers and Klaus' (2016, Theorem 1) axiomatization of DA to many-to-one school choice problems.

In matching problems with externalities, an agent cares not only about her match but also about the distribution of others. This literature was initiated by Sasaki and Toda (1996), who studied stability concepts in marriage markets. In recent years, several authors have extended the analysis to many-to-one matching models and other allocation problems. They have introduced restrictions in preference domains to specify the type of externalities and to ensure the existence of stable outcomes.[6] In the context of many-to-one matching problems, Dutta and Massó (1997) studied a two-sided model with workers and firms where workers' preferences are lexicographic. When workers' preferences over firms dictate their overall preferences over firm-colleague pairs, they show that the set of stable matchings is non-empty (cf., Fonseca-Mairena and Triossi, 2023). Complementing this approach, Echenique and Yenmez (2007) assume that workers have general preferences over colleagues and present an algorithm that produces a set of allocations containing all stable matchings, if they exist. Moreover, Revilla (2007) and Bykhovskaya (2020) determine subdomains of preferences over colleagues where a stable matching always exists. More recently, Pycia and Yenmez (2023) studied a hybrid model that allows for general types of externalities, including school choice problems as a particular case. They demonstrate that a stable matching exists as long as externalities affect students' choice rules in such a way that a *sustitutability condition* is satisfied, which is also necessary for stability to some extent.

[6]The works of Bando, Kawasaki, and Muto (2016) and Pycia and Yenmez (2023) provide excellent surveys on the evolution of this literature.

Although Dutta and Massó (1997) demonstrate that the set of stable matchings when preferences are school-lexicographic is the same as without externalities, Duque and Torres-Martínez (2023) show that a stable and strategy-proof mechanism may not exist. We contribute to this strand of the literature by identifying a new preference domain where this incompatibility does not hold. In particular, when students first consider the school to which they are assigned and then only their classmates, DA induces the only stable and strategy-proof mechanism. Moreover, it is the local non-bossiness of DA that underlies this result.

The rest of the paper is organized as follows. Section 2 describes the classical school choice problem and the concepts of local non-bossiness and local group strategy-proofness. In Section 3 we show that DA is locally non-bossy. Section 4 provides an axiomatic characterization of DA. In Section 5 we introduce externalities in our model, and we show that DA induces a stable and strategy-proof mechanism in the domain of school-lexicographic preferences over colleagues. Section 6 contains the final remarks. Omitted proofs and examples are left to the appendices.

2. Model

Let N be a set of students and S a set of schools. Each $s \in S$ has a priority order $\succ_s$ for students, and a capacity $q_s \geq 1$. Every $i \in N$ is characterized by a complete, transitive, and strict preference relation P_i defined on $S \cup \{s_0\}$, where s_0 represents an outside option. Denote by R_i the weak preference induced by P_i and refer to a school s as admissible when sP_is_0. Given $\succ = (\succ_s)_{s\in S}$, $q = (q_s)_{s\in S}$, and $P = (P_i)_{i\in N}$, we refer to $[N, S, \succ, q]$ as a **school choice context** and to $[N, S, \succ, q, P]$ as a **school choice problem**.

A matching is a function $\mu : N \to S \cup \{s_0\}$ that assigns students to schools in such a way that at most q_s students are assigned to $s \in S$. When $\mu(i) = s_0$, student i is not assigned to any school. Let $\mathcal{M}$ be the set of matchings and $\mu(s) = \{i \in N : \mu(i) = s\}$ be the set of students that μ assigns to $s \in S \cup \{s_0\}$. A matching μ is **individually rational** if no student i prefers s_0 to $\mu(i)$. Moreover,

μ is **stable** if it is individually rational and there is no $(i, s) \in N \times S$ such that $sP_i\mu(i)$ and either $|\mu(s)| < q_s$ or $i \succ_s j$ for some $j \in \mu(s)$.

A mechanism is a function that associates a matching to each school choice problem. The mechanism designer is assumed to know the school choice context $[N, S, \succ, q]$ and that students' preferences belong to the preference domain $\mathcal{P} = \mathcal{L}^{|N|}$, where $\mathcal{L}$ is the set of strict linear orders defined on $S \cup \{s_0\}$. Given $P = (P_i)_{i \in N} \in \mathcal{P}$ and $C \subseteq N$, let $P_C = (P_i)_{i \in C}$ and $P_{-C} = (P_i)_{i \notin C}$.

When a school choice context $[N, S, \succ, q]$ is fixed, we will include only the preferences of the students as the argument of a mechanism $\Phi : \mathcal{P} \to \mathcal{M}$, that is $\Phi(P)$ instead of $\Phi(N, S, \succ, q, P)$. Denote by $\Phi_i(P_i, P_{-i})$ the assignment of i when she declares preferences P_i and the other students declare $P_{-i} = (P_j)_{j \neq i}$. Similarly, $\Phi_s(P)$ denotes the set of students assigned to $s \in S \cup \{s_0\}$.

A mechanism Φ is **stable (individually rational)** if for all $P \in \mathcal{P}$ the matching $\Phi(P)$ is stable (individually rational). Consider the following properties:

- Φ is **strategy-proof** if there are no $i \in N$, $P \in \mathcal{P}$, and $P'_i \in \mathcal{L}$ such that $\Phi_i(P'_i, P_{-i}) P_i \Phi_i(P)$.
- Φ is **non-bossy** if for all $i \in N$, $P \in \mathcal{P}$, and $P'_i \in \mathcal{L}$, $\Phi_i(P) = \Phi_i(P'_i, P_{-i})$ implies that $\Phi(P) = \Phi(P'_i, P_{-i})$.
- Φ is **group strategy-proof** if there are no $P \in \mathcal{P}$, $C \subseteq N$, and $P'_C \in \mathcal{L}^{|C|}$ such that:
  - For some $i \in C$, $\Phi_i(P'_C, P_{-C})\, P_i\, \Phi_i(P)$.
  - For each $j \in C$, $\Phi_j(P'_C, P_{-C})\, R_j\, \Phi_j(P)$.

Strategy-proofness requires that no one has incentives to misreport her preferences, while non-bossiness ensures that a change in the preferences of a student cannot modify the schools of others without changing her own. Under group strategy-proofness no coalition of students can benefit by misrepresenting their preferences.

When a *non-bossy* mechanism is implemented, no one can change the assignment of other students without changing her own.[7] If instead of

[7] We refer to a mechanism as *bossy* when it is not non-bossy.

considering the change in the assignment of any student, we focus only on those who are assigned to the same school as the one who change her preferences, we have a *local* version of non-bossiness.

Definition 1. *A mechanism Φ is **locally non-bossy** if for all $i \in N$, $P \in \mathcal{P}$, $P'_i \in \mathcal{L}$, and $s \in S \cup \{s_0\}$, $\Phi_i(P) = \Phi_i(P'_i, P_{-i}) = s$ implies that $\Phi_s(P) = \Phi_s(P'_i, P_{-i})$.*

When a mechanism is locally non-bossy, no student can modify her classmates without also changing the school to which she is assigned. The definition contemplates a situation where the student is unassigned. In this case, the student cannot modify the set of unassigned students without getting a seat in a school. This implies that local non-bossiness is non-trivial even in a one-to-one matching problem.

When a mechanism is not group strategy-proof, a coalition of students can benefit by manipulating their preferences. The members of this coalition may originally be assigned to different schools. When we restrict the coalition of students to be originally assigned to the same school, we have a *local* version of group strategy-proofness.

Definition 2. *A mechanism Φ is **locally group strategy-proof** if there are no $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, $C \subseteq \Phi_s(P)$, and $P'_C \in \mathcal{L}^{|C|}$ such that:*

- *For some $i \in C$, $\Phi_i(P'_C, P_{-C})\, P_i\, \Phi_i(P)$.*
- *For each $j \in C$, $\Phi_j(P'_C, P_{-C})\, R_j\, \Phi_j(P)$.*

When a mechanism is locally group strategy-proof, no coalition of *classmates* can manipulate it to improve the welfare of at least one of its members.

In Appendix A we analyze the relationships between local non-bossiness, local group strategy-proofness, and other incentive properties (the Figure 1 summarizes our results). Specifically, we demonstrate that local non-bossiness is independent of both stability and strategy-proofness, while local group strategy-proofness is weaker than group strategy-proofness and stronger than strategy-proofness. Furthermore, any locally non-bossy and strategy-proof mechanism is locally group strategy-proof (Lemma 1), but a locally group strategy-proof mechanism might violate local non-bossiness. This last result contrasts with the well-known equivalence between group strategy-proofness and the combination of non-bossiness and strategy-proofness (Papái, 2000). Additionally, we show in Lemma 2 that any locally non-bossy and strategy-proof mechanism satisfies a local notion of group non-bossiness (Afacan, 2012), and in Lemma 3 that local non-bossiness is independent of weak non-bossiness (Bando and Imammura, 2016).[8]

3. Local non-bossiness of DA

Given a school choice context $[N, S, \succ, q]$, let DA : $\mathcal{P} \rightarrow \mathcal{M}$ be the **student-optimal stable mechanism**, namely the rule that associates to each school choice problem the outcome of the deferred acceptance algorithm when students make proposals.

Student-Proposing Deferred Acceptance Algorithm

Given $[N, S, (\succ_s, q_s)_{s \in S}, (P_i)_{i \in N}]$,

Step 1: Each student i proposes to the most preferred admissible school according to P_i, if any. Among those that have proposed to it, each school s provisionally accepts the q_s-highest ranked students according to $\succ_s$.

Step $k \geq 2$: Each student i who has not been provisionally accepted in step $k-1$ proposes to the most preferred admissible school according to P_i, among those to which she has not previously proposed, if any. Each school s provisionally accepts the q_s-highest ranked students according

[8]A notion related to non-bossiness is *consistency* (Ergin, 2002). A mechanism is consistent if after removing some students with their seats, the rest of the students remain in the same school when the mechanism is applied to the reduced market. It is easy to formulate a local notion of consistency and to show that a locally consistent mechanism is locally non-bossy.

to $\succ_s$, among those who have proposed to it in this step or were provisionally accepted by it in step $k-1$.

The algorithm terminates at the step in which no proposals are made. Provisional acceptances become definitive then.

For any preference profile $P \in \mathcal{P}$, the matching $\mathrm{DA}(P)$ assigns to each student the most preferred alternative in $S \cup \{s_0\}$ that she can reach in a stable matching under P (Gale and Shapley, 1962; Roth, 1985). Moreover, the mechanism DA is strategy-proof (Dubins and Freedman, 1981; Roth, 1982).

However, when DA is implemented a student can modify the assignment of others without affecting her own (Ergin, 2002). Hence, one may wonder if there is a limit on the impact of these changes. In particular, under DA, is it possible for a student to affect the set of students assigned to her school by reporting different preferences, without changing her school ? As the following theorem shows, the answer is negative.

Theorem 1. *The student-optimal stable mechanism is locally non-bossy.*

To get an intuitive idea of the proof of Theorem 1, let $\mu = \mathrm{DA}(P)$ and $\mu' = \mathrm{DA}(P'_i, P_{-i})$ be distinct matchings that give the same assignment to student i, namely $\bar{s} \equiv \mu(i) = \mu'(i)$.

By appealing to the Rural Hospital Theorem, we can show that students who receive different assignments in μ and μ' are in schools that fill their quotas in both matchings. Hence, when we compare μ' to μ, each student who changes assignments displaces another. These movements induce *cycles* of students $(i_1, \ldots, i_r)$, where each i_k receives in μ' the school that i_{k+1} had in μ (modulo r). If there exists a student who prefers μ' to μ,[9] the stability of μ and μ' ensures that at least one of these cycles is a *μ-improving cycle*—that is, a cycle $(i_1, \ldots, i_r)$ in which every i_k prefers $\mu(i_{k+1})$ to $\mu(i_k)$ (modulo r).

[9] If this does not occur, we can maintain our arguments by simply swapping the roles of P and (P'_i, P_{-i}).

We define a graph G by letting each student j who prefers μ' to μ point to those who are assigned in μ to the school $\mu'(j)$. Previous arguments ensure that G has a μ-improving cycle. Since μ is the student-optimal stable matching under P, any μ-improving cycle necessarily induces justified envy. By allowing students in G to block the improving cycles contained in it, we construct a new μ-improving cycle $(i_1^*, \ldots, i_r^*)$ in which justified envy is limited to students outside G—that is, to those who are assigned in μ to schools that, in μ', do not receive anyone who prefers μ' to μ.[10] Moreover, since students in $N \setminus \{i\}$ do not change their preferences, we can show that only i blocks $(i_1^*, \ldots, i_r^*)$. Hence, the students in $\mu'(\bar{s})$ weakly prefer μ to μ'. This result implies that either $\mu'(\bar{s}) \subseteq \mu(\bar{s})$ or $\mu(\bar{s}) \subseteq \mu'(\bar{s})$. Finally, to conclude that $\mu(\bar{s}) = \mu'(\bar{s})$, we invoke the Rural Hospital Theorem to ensure that any school with different entering classes in μ and μ' always fills its quota.

Proof of Theorem 1. Fix a school choice context $[S, N, \succ, q]$ and w.l.o.g consider student 1 and preference profiles $(P_1, P_{-1}), (P_1', P_{-1}) \in \mathcal{P}$ such that $\bar{s} \equiv \mathrm{DA}_1(P_1, P_{-1}) = \mathrm{DA}_1(P_1', P_{-1})$ and $\mathrm{DA}(P_1, P_{-1}) \neq \mathrm{DA}(P_1', P_{-1})$. We will prove that $\mathrm{DA}_{\bar{s}}(P_1, P_{-1}) = \mathrm{DA}_{\bar{s}}(P_1', P_{-1})$.

When $\bar{s} = s_0$, the result follows from the fact that no student who is not assigned a seat in $\mathrm{DA}(P_1, P_{-1})$ can be assigned a seat in $\mathrm{DA}(P_1', P_{-1})$ (see Lemma 4 in Appendix B).

Assume that $\bar{s} \neq s_0$. Let $\mu = \mathrm{DA}(P_1, P_{-1})$ and $\mu' = \mathrm{DA}(P_1', P_{-1})$. Notice that, without loss of generality, we can assume that the set I of students who prefer μ' to μ is non-empty.[11]

Let $G = (V, E)$ be the graph with nodes $V = I \cup \{j : \mu(j) = \mu'(i) \text{ for some } i \in I\}$ and directed edges $E = \{[i, j] : i \in I, \mu(j) = \mu'(i)\}$. Hence, the nodes of G are the students who prefer μ' to μ, together with those who are assigned in μ to the schools to which students in I are assigned in μ'. Also, there is an edge between every $i \in I$ and all the students who are assigned to $\mu'(i)$ in μ.

[10] The μ-improving cycle $(i_1^*, \ldots, i_r^*)$ is not necessarily present in G.

[11] Since $\mu(1) = \mu'(1)$ and $\mu \neq \mu'$, when I is an empty set there is at least one student who prefers μ to μ'. Hence, our arguments hold by swapping the roles of (P_1, P_{-1}) and (P_1', P_{-1}).

Consider the following concepts:

- A *μ-improving cycle* is a tuple of different students $(i_1, \ldots, i_r)$ such that

$$\mu(i_{l+1}) P_{i_l} \mu(i_l), \qquad \forall l \in \{1, \ldots, r\} \text{ [modulo } r\text{]}.$$

- A student *k μ-blocks* $[i,j]$ when $\mu(j) P_k \mu(k)$ and $k \succ_{\mu(j)} i$.
- A student *k blocks a μ-improving cycle* $(i_1, \ldots, i_r)$ when she μ-blocks $[i_l, i_{l+1}]$ for some $l \in \{1, \ldots, r\}$ [modulo r].

The following properties are satisfied:

(i) Each $i \in I$ is part of a μ-improving cycle contained in G (see Lemma 5 in Appendix B).

(ii) Each node in G has a positive in-degree.

By construction, each node in $V \setminus I$ has a positive in-degree. Moreover, the property (i) ensures that each student $i \in I$ is part of a μ-improving cycle contained in G. Hence, there exists $j \in V$ such that $[j, i] \in E$.

Let $G' = (V, E')$ be the *multigraph* obtained from G through the following edge-replacement procedure: given a student $j \in N$, substitute each edge $[i,j] \in E$ for which

$$I[i,j] \equiv \{k \in V : k \ \mu\text{-blocks } [i,j]\} \neq \emptyset$$

by the directed edge $[\overline{k}, j]$, where $\overline{k}$ is the student with the highest priority at school $\mu(j)$ among those in $I[i,j]$. Notice that, when $[\overline{k}, j] \notin E$, its inclusion in E' is feasible because $\overline{k} \in I[i,j] \subseteq V$.

We make the following remarks regarding graph G'.

(1) *The graph G' has a cycle $(i_1^*, \ldots, i_r^*)$.*

It follows from (ii) that all vertices in V have a positive in-degree in G'. Indeed, when E' is constructed from E, if some directed edge $[i,j]$ is deleted, then an edge who points to j is included. As a consequence, G' has a cycle $(i_1^*, \ldots, i_r^*)$.

(2) *Any cycle in G' is a μ-improving cycle.*

Remember that, if a student k μ-blocks $[i,j] \in E$, then $\mu(j) P_k \mu(k)$. Hence, each $[k,j] \in E' \setminus E$ satisfies $\mu(j) P_k \mu(k)$. Since the definition of

E ensure that any $[k,j] \in E' \cap E$ satisfies $\mu(j) P_k \mu(k)$, we conclude that any cycle in G' is a μ-improving cycle.

(3) *Any cycle in G' cannot be blocked by any student in V.*
It is a direct consequence of the construction of the set of edges E'.

(4) *The μ-improving cycle $(i_1^*, \ldots, i_r^*)$ must be blocked by student* 1.
Since μ is the student-optimal stable matching under (P_1, P_{-1}), the μ-improving cycle $(i_1^*, \ldots, i_r^*)$ must be blocked by some student in $N \setminus V$.
For each $[i,j] \in E'$, we have that $[i,j] \in E$ or $[i,j] \in E' \setminus E$. Hence, $[i,j] \in E$ or there exists $[z,j] \in E$ that is μ-blocked by i. This implies that either $\mu(j) = \mu'(i)$ or there exists $z \in I$ such that [$\mu(j) = \mu'(z)$, $\mu(j) P_i \mu(i)$, and $i \succ_{\mu(j)} z$]. As a consequence, when a student k μ-blocks an edge $[i,j] \in E'$, we have that either [$\mu'(i) P_k \mu(k)$ and $k \succ_{\mu'(i)} i$] or [$\mu'(z) P_k \mu(k)$ and $k \succ_{\mu'(z)} z$] .
We claim that no student $k \neq 1$ can block an edge in E'. By contradiction, suppose that $k \neq 1$ μ-blocks an edge $[i,j] \in E'$. It follows from the arguments above and the stability of μ' that there are two possible scenarios:
- If $\mu'(i) P_k \mu(k)$ and $k \succ_{\mu'(i)} i$, then $\mu'(k) R_k \mu'(i)$. Since $\mu'(i) P_k \mu(k)$, the transitivity of P_k ensures that $k \in I$. Hence, $k \in V$, which contradicts the property (3).
- If $\mu'(z) P_k \mu(k)$ and $k \succ_{\mu'(z)} z$, then $\mu'(k) R_k \mu'(z)$. Since $\mu'(z) P_k \mu(k)$, the transitivity of P_k ensures that $k \in I$. Hence, $k \in V$, which contradicts the property (3).

We conclude that $(i_1^*, \ldots, i_r^*)$ must be blocked by student 1.

We finish the proof by showing that $\mu(\bar{s}) = \mu'(\bar{s})$.

Since $(i_1^*, \ldots, i_r^*)$ is blocked by student 1, we have that $1 \notin V$. Thus, the school $\bar{s}$ does not receive in μ' students who prefer μ' to μ.

We claim that $\mu'(\bar{s}) \subseteq \mu(\bar{s})$ or $\mu(\bar{s}) \subseteq \mu'(\bar{s})$. Indeed, there are two possible scenarios:

- If no one prefers μ to μ', then everyone in $\mu'(\bar{s})$ is indifferent between μ' to μ, which implies that $\mu'(\bar{s}) \subseteq \mu(\bar{s})$.

- If at least one student prefers μ to μ', then we can repeat all the previous arguments swapping the roles of (P_1, P_{-1}) and (P_1', P_{-1}) in order to show that $\bar{s}$ does not receive in μ students who prefers μ to μ'. Notice that, if there exists $i \in \mu(\bar{s}) \setminus \mu'(\bar{s})$, then i belongs to I. Hence, the property (i) guarantees that i is part of a μ-improving cycle contained in G, which ensures that there is $k \in I$ such that $[k, i] \in E$. This implies that $1 \in V$, because $\mu'(k) = \mu(i) = \bar{s} = \mu(1)$. A contradiction. Hence, $\mu(\bar{s}) \setminus \mu'(\bar{s})$ is an empty set, which implies that $\mu(\bar{s}) \subseteq \mu'(\bar{s})$.

Finally, since Lemma 4 guarantees that $|\mu(s)| = |\mu'(s)|$ for all $s \in S$ such that $\mu(s) \neq \mu'(s)$, we conclude that $\mu(\bar{s}) = \mu'(\bar{s})$. □

As Theorem 1 demonstrates, when the DA mechanism is implemented, no student can modify her preferences to change her classmates without changing her school. But what happens when multiple students, *all assigned to the same school*, change their preferences? Can they change their classmates without changing their school? The answer is negative, and not only for DA, but also for any locally non-bossy and strategy-proof mechanism (see Lemma 2).

Remark 1. Let $[N, S, \succ, q]$ be such that $|N| = \sum_{s \in S} q_s$, and assume that every student considers all schools admissible. We claim that when DA is implemented, a student who misreports her preferences and changes her assignment, also modifies all her colleagues. More formally, if $\mathcal{P}^* \subseteq \mathcal{P}$ is the preference domain in which everyone considers all schools admissible and $\text{Coll}_i(P) = \{j \in N : j \neq i, \text{DA}_j(P) = \text{DA}_i(P)\}$ is the set of colleagues of student i in $\text{DA}(P)$, then the following property holds (see Lemma 6):

- For all $i \in N$, $P \in \mathcal{P}^*$, and $P_i' \in \mathcal{L}$ such that $(P_i', P_{-i}) \in \mathcal{P}^*$,

$$\text{DA}_i(P) \neq \text{DA}_i(P_i', P_{-i}) \implies \text{Coll}_i(P) \cap \text{Coll}_i(P_i', P_{-i}) = \emptyset.$$

In particular, this property implies that the converse of local non-bossiness is satisfied in this context: if the colleagues of some student do not change when her preferences are modified, then her school does not change too. □

Notice that, DA is not the only locally non-bossy and stable mechanism that can be defined in $\mathcal{P}$. Indeed, the *school-optimal* stable mechanism also satisfies these properties, because it is non-bossy (Afacan and Dur, 2017). Furthermore, the arguments made in the proof of Theorem 1 can be easily adapted to provide a direct proof that the school-optimal stable mechanism is locally non-bossy. Indeed, since this mechanism assigns each student to the worst school she can reach in a stable matching, it is sufficient to repeat the proof focusing on *μ-worsening cycles* (i.e., tuples $(i_1, \dots, i_r)$ such that $(i_r, \dots, i_1)$ forms a μ-improving cycle).

It is well-known that DA is the only mechanism in $\mathcal{P}$ that is both stable and strategy-proof (Alcalde and Barberà, 1994). Since locally group strategy-proofness is stronger than strategy-proofness, and locally non-bossiness and strategy-proofness ensure locally group strategy-proofness (Lemma 1), the following result is a direct consequence of Theorem 1.

Corollary 1. DA *is the only mechanism that is stable and locally group strategy-proof.*

4. A CHARACTERIZATION OF DA VIA LOCAL NON-BOSSINESS

In this section, we provide a characterization of DA using axioms that do not involve priorities. Our result extends the characterization of DA given by Ehlers and Klaus (2016, Theorem 1) from one-to-one matching problems to many-to-one contexts.

To maintain the generality of Ehlers and Klaus' (2016) context, we analyze a variable-population school choice model. Hence, given a population $\overline{N}$ of students and a set S of schools with capacities $q = (q_s)_{s \in S}$, we consider mechanisms that depend only on the set $N \subseteq \overline{N}$ of students in the market and their preferences $(P_i)_{i \in N}$.

To simplify notations, we consider $\mathcal{P} = \mathcal{L}^{\overline{N}}$ as the preference domain, although only the preferences of the agents in the market will be relevant at the moment of determine a matching. Let $\mathcal{N}$ be the collection of non-empty subsets of $\overline{N}$. Given $N \in \mathcal{N}$, let $\mathcal{M}(N)$ be the set of matchings compatible with $[N, S, q]$. That is, $\mathcal{M}(N)$ is the family of functions $\mu : N \to S \cup \{s_0\}$ such that

$|\{i \in N : \mu(i) = s\}| \leq q_s$ for all $s \in S$. Let $\mathcal{M}^* = \bigcup_{N \in \mathcal{N}} \mathcal{M}(N)$. For each $i \in \overline{N}$ and $P_i \in \mathcal{L}$, denote by $A(P_i) = \{s \in S : sP_i s_0\}$ the set of schools that are admissible under P_i.

A mechanism is a function $\Phi : \mathcal{N} \times \mathcal{P} \to \mathcal{M}^*$ that associates to each $(N, P) \in \mathcal{N} \times \mathcal{P}$ a matching in $\mathcal{M}(N)$ that only depends on $[N, S, q, (P_i)_{i \in N}]$.

Consider the following properties:

- Φ is **weakly non-wasteful** when for all $N \in \mathcal{N}$, $i \in N$, $P \in \mathcal{P}$, and $s \in S$, if $sP_i\Phi_i(N, P)$ and $\Phi_i(N, P) = s_0$, then $|\Phi_s(N, P)| = q_s$.
- Φ is **population-monotonic** if for all $N, N' \in \mathcal{N}$ such that $N \subseteq N'$, $i \in N$, and $P \in \mathcal{P}$ we have that $\Phi_i(N, P) R_i \Phi_i(N', P)$.

Under weak non-wastefulness, no unassigned student prefers a school that did not fill its places. Population-monotonicity guarantees that when the set of students enlarges, those who were initially present are (weakly) worse off.

Definition 3. *A mechanism Φ is **weakly WrARP** when for all $N, N' \in \mathcal{N}$, $P \in \mathcal{P}$, and $s \in S$ such that $|N| = |N'| = q_s + 1$ and $A(P_k) = \{s\}$ for all $k \in N \cup N'$,*

$$\Big[\, i, j \in N \cap N', \quad i \in \Phi_s(N, P), \quad j \in \Phi_s(N', P) \setminus \Phi_s(N, P) \,\Big] \implies \big[\, i \in \Phi_s(N', P) \,\big].$$

Weak WrARP allows us to construct schools' choice functions which are consistent with a priority order (Chambers and Yenmez, 2018). Intuitively, the property considers a situation where two students, i and j, can be chosen from two different sets. If i is chosen but not j when both are available, then whenever j is chosen, i should also be chosen if she is available.

Definition 4. *A mechanism Φ is **weakly locally non-bossy** if for all $N \in \mathcal{N}$, $i \in N$, $P \in \mathcal{P}$, $P'_i \in \mathcal{L}$, and $s \in S$, we have that $\Phi_i(N, P) = \Phi_i(N, (P'_i, P_{-i})) = s$ implies that $\Phi_s(N, P) = \Phi_s(N, (P'_i, P_{-i}))$.*

Weak local non-bossiness restricts local non-bossiness to the set of (real) schools S.

Given a priority profile $\succ$, let $\text{DA}^{\succ}$ be the mechanism that associates to each $(N,P) \in \mathcal{N} \times \mathcal{P}$ the student-optimal stable matching of $[N, S, \succ^N, q, (P_i)_{i\in N}]$, where $\succ^N$ is the priority profile induced by $\succ$.[12] When each school has only one available seat, Ehlers and Klaus (2016) demonstrate that a mechanism satisfies individual rationality, weak non-wastefulness, population-monotonicity, and strategy-proofness if and if it is equal to $\text{DA}^{\succ}$ for some priority profile $\succ$. Furthermore, they show that this characterization does not hold in the general case in which schools have non-unit capacities.

The following result shows that the four axioms of Ehlers and Klaus (2016) jointly with weak WrARP and weak local non-bossiness fully characterize the family of mechanisms $\text{DA}^{\succ}$, where $\succ$ is an arbitrary priority profile. Notice that the two new axioms are trivially satisfied when each school has only one available seat.

Theorem 2. *A mechanism $\Phi : \mathcal{N} \times \mathcal{P} \to \mathcal{M}^*$ is individually rational, weakly non-wasteful, population-monotonic, strategy-proof, weakly WrARP, and weakly locally non-bossy if and only if $\Phi = \text{DA}^{\succ}$ for some priority profile $\succ$.*

The proof is given in the Appendix B.

In our characterization of DA, weak local non-bossiness is crucial to manage the schools with more than one seat available. Indeed, Example 1 of Ehlers and Klaus (2016), which we reproduce below, allows us to describe a mechanism that satisfies all the axioms of Theorem 2 but weak local non-bossines.

Example 1. Let $\overline{N} = \{1,2,3,4\}$, $S = \{s_1, s_2\}$, and $(q_{s_1}, q_{s_2}) = (2,1)$. Consider the priority orders $\succ = (\succ_{s_1}, \succ_{s_2})$ and $\succ' = (\succ'_{s_1}, \succ_{s_2})$ characterized by $\succ_{s_1}$: $1,2,3,4$, $\succ_{s_2}$: $1,2,3,4$, and $\succ'_{s_1}$: $1,2,4,3$. Denote by $top(P_i)$ the most preferred alternative of student i when her preferences are $P_i \in \mathcal{L}$. Let $\Omega : \mathcal{N} \times \mathcal{P} \to \mathcal{M}^*$ be the

[12] That is, if $\succ = (\succ_s)_{s\in S}$, the induced priority profile $\succ^N = (\succ^N_s)_{s\in S}$ is such that, for all $i, j \in N$ and $s \in S$, $i \succ^N_s j$ whenever $i \succ_s j$.

mechanism such that:

$$\Omega(N,P) = \begin{cases} \mathrm{DA}^{\succ'}(N,P), & \text{when } \{1,2\} \subseteq N \text{ and } top(P_1) = top(P_2) = s_2; \\ \mathrm{DA}^{\succ}(N,P), & \text{in another case.} \end{cases}$$

Ehlers and Klaus (2016, Example 1) show that Ω is individually rational, weakly non-wasteful, population-monotonic, and strategy-proof. Furthermore, Ω is weakly WrARP, because for each profile of preferences the school s_1 chooses students from a set following a priority order.

However, Ω is weakly locally bossy. Indeed, when preferences are given by

$$P_1 : s_2, s_0, s_1, \qquad P_2 : s_1, s_0, s_2, \qquad P_3 : s_1, s_0, s_2 \qquad P_4 : s_1, s_0, s_2,$$

we have that $\Omega(\overline{N}, P) = \mathrm{DA}^{\succ}(\overline{N}, P) = ((1, s_2), (2, s_1), (3, s_1), (4, s_0))$. If $P'_2 : s_2, s_1, s_0$, $\Omega(\overline{N}, (P'_2, P_{-2})) = \mathrm{DA}^{\succ'}(\overline{N}, (P'_2, P_{-2})) = ((1, s_2), (2, s_1), (3, s_0), (4, s_1))$. Therefore, the student 2 can change her classmates without modify her school.[13] □

A priority profile $\succ$ has an **Ergin-cycle** when for some $s, s' \in S$ and $i, j, k \in \overline{N}$ we have that $i \succ_s j \succ_s k \succ_{s'} i$ and there are disjoint sets $N_s, N_{s'} \subseteq \overline{N} \setminus \{i, j, k\}$, with $|N_s| = q_s - 1$ and $|N_{s'}| = q_{s'} - 1$, such that j has lower priority in s than everyone in N_s and i has lower priority in s' than everyone in $N_{s'}$. $\succ$ is **acyclic** when it does not have Ergin-cycles.

Corollary 2. *A mechanism $\Phi : \mathcal{N} \times \mathcal{P} \to \mathcal{M}^*$ is individually rational, weakly non-wasteful, population-monotonic, group strategy-proof, and weakly WrARP if and only if $\Phi = \mathrm{DA}^{\succ}$ for some acyclic priority profile $\succ$.*

Proof. Group strategy-proofness ensures that strategy-proofness and non-bossiness hold (Papái, 2000). Hence, the Theorem 2 implies that any mechanism Φ satisfying the five axioms described above coincides with $\mathrm{DA}^{\succ}$, for some priority profile $\succ$. In particular, as $\mathrm{DA}^{\succ}$ is group strategy-proof, $\succ$ is

[13] This argument also shows that Ω is not group strategy-proof. Indeed, the coalition $\{2, 4\}$ can report preferences (P'_2, P_4) to improve the assignment of 4 without hurting 2. Hence, the same example allows us to show that group strategy-proofness is a critical axiom in Corollary 2.

acyclic (Ergin, 2002). Reciprocally, given an acyclic priority profile $\succ$, it follows from Ergin (2002, Theorem 1) that $DA^{\succ}$ is group strategy-proof (and Theorem 2 ensures that $DA^{\succ}$ satisfies the other axioms). □

5. Mechanisms under preferences over colleagues

Most of the literature in school choice assumes that students only care about the school to which they are assigned. However, it is unrealistic that students do not care about the assignment of others. In particular, it has been empirically documented that the identity of classmates is an important consideration when students are choosing a school (Rothstein, 2006; Abdulkadiroğlu et al., 2020; Allende, 2021; Che et al., 2022; Beuermann et al., 2023; Cox et al., 2023).

When students have arbitrary preferences over the set of matchings, many of the results in the literature break down. Specifically, a stable matching may not exist, *even when students only care about the identity of the other students assigned to her school* (Echenique and Yenmez, 2007). One way to overcome this problem is to restrict the domain of preferences. For instance, assuming that students have *school-lexicographic preferences*, meaning that they are primarily concerned with the school to which they are assigned (Sasaki and Toda, 1996; Dutta and Massó, 1997). In this domain, some of the standard results are recovered, such as the existence of a stable matching. However, a stable and strategy proof mechanism may not exist (Duque and Torres-Martínez, 2023). One may wonder if there is a subdomain of school-lexicographic preferences where stability and strategy-proofness are compatible. In this section, we show that within the subdomain of school-lexicographic preferences where only the classmates matter, the local non-bossiness of DA implies that this rule induces a stable and strategy-proof mechanism.

We first define the domain of school-lexicographic preferences, and then the subdomain where there exists a stable and strategy-proof mechanism.

Definition 5. *Given a school choice context $[N, S, \succ, q]$, a* ***school-lexicographic preference*** *for student $i \in N$ is a complete and transitive preference relation $\trianglerighteq_i$ defined on $\mathcal{M}$ such that, for any pair of matchings $\mu, \eta \in \mathcal{M}$, the following properties hold:*

- *If $\mu(i) \neq \eta(i)$, then either $\mu \rhd_i \eta$ or $\eta \rhd_i \mu$, where $\rhd_i$ is the strict part of $\trianglerighteq_i$.*
- *If $\mu \rhd_i \eta$ and $\mu(i) \neq \eta(i)$, then $\mu' \rhd_i \eta'$ for all matchings $\mu', \eta' \in \mathcal{M}$ such that $\mu'(i) = \mu(i)$ and $\eta'(i) = \eta(i)$.*

Let $\mathcal{D}$ be the set of preference profiles $\trianglerighteq = (\trianglerighteq_i)_{i \in N}$ satisfying these properties.

The first condition states that a student cannot be indifferent between two matchings where she is assigned to different schools. By the second condition, if a student prefers a matching μ over η, and she is assigned to different schools in these two matchings, then she should prefer every matching where she is assigned to the same school as in μ to every matching where she is assigned to the same school as in η.

Notice that, in the preference domain $\mathcal{D}$ no restrictions are imposed on the ranking of two matchings in which a student is assigned to the same school. Let $P(\trianglerighteq) = (P_i(\trianglerighteq))_{i \in N} \in \mathcal{P}$ be the preferences over schools induced by $\trianglerighteq = (\trianglerighteq_i)_{i \in N} \in \mathcal{D}$ through the following rule: given $s, s' \in S \cup \{s_0\}$, $s\, P_i(\trianglerighteq)\, s'$ if and only if there exist $\mu, \mu' \in \mathcal{M}$ such that $\mu(i) = s$, $\mu'(i) = s'$, and $\mu \rhd_i \mu'$. The second property of Definition 5 ensures that the preferences $P(\trianglerighteq)$ are well-defined.

One can naturally extend the concepts of stability and strategy-proofness to the domain of school-lexicographic preferences using the induced preferences over schools. Given $\mathcal{D}' \subseteq \mathcal{D}$, a **stable mechanism** associates to each preference profile $\trianglerighteq \in \mathcal{D}'$ a stable matching of $[N, S, \succ, q, \trianglerighteq]$. A mechanism Γ is **strategy-proof** in $\mathcal{D}'$ when there is no student i such that $\Gamma_i(\trianglerighteq_i', \trianglerighteq_{-i}) \rhd_i \Gamma_i(\trianglerighteq)$ for some $\trianglerighteq$ and $(\trianglerighteq_i', \trianglerighteq_{-i})$ in $\mathcal{D}'$.

School-lexicographic preferences have no effects on stability, because the school choice problems $[N, S, \succ, q, \trianglerighteq]$ and $[N, S, \succ, q, P(\trianglerighteq)]$ have the same set of stable matchings (Sasaki and Toda, 1996; Dutta and Massó, 1997; Fonseca-Mairena and Triossi, 2023). However, there are strong effects on incentives as there may not exist a stable and strategy-proof mechanism (Duque and Torres-Martínez, 2023).

Within the domain of school-lexicographic preferences, let us consider a scenario where students are solely concerned about the composition of the school to which they are assigned. Consequently, a student will be indifferent between two matchings when she is assigned to the same school with the same classmates. This is the idea of the following subdomain of preferences.

Definition 6. *Given $[N, S, \succ, q]$, the domain of* ***school-lexicographic preferences over colleagues*** *$\mathcal{D}_c \subseteq \mathcal{D}$ is the set of profiles $(\trianglerighteq_i)_{i\in N}$ such that $\mu \triangleright_i \eta$ and $\mu(i) = \eta(i) = s$ imply that $\mu(s) \neq \eta(s)$.*

In words, when $(\trianglerighteq_i)_{i\in N}$ belongs to $\mathcal{D}_c$, each student i strictly prefers μ to η only when she is assigned to different schools or to the same school with different classmates.

The next result shows that there is a stable and strategy-proof mechanism in $\mathcal{D}_c$, which is the student optimal stable mechanism applied to $[N, S, \succ, q, P(\trianglerighteq)]$.

Theorem 3. *In any school choice context $(S, N, \succ, q)$, the mechanism $\overline{\mathrm{DA}} : \mathcal{D}_c \to \mathcal{M}$ defined by $\overline{\mathrm{DA}}(\trianglerighteq) = \mathrm{DA}(P(\trianglerighteq))$ is stable and strategy-proof.*

Proof. Fix $\trianglerighteq \in \mathcal{D}_c$. The stability of $\overline{\mathrm{DA}}(\trianglerighteq)$ is a consequence of the fact that $\mathrm{DA} : \mathcal{P} \to \mathcal{M}$ is stable, because $[N, S, \succ, q, \trianglerighteq]$ and $[N, S, \succ, q, P(\trianglerighteq)]$ have the same stable matchings. By contradiction, assume that $\overline{\mathrm{DA}}$ is not strategy-proof in $\trianglerighteq$. Hence, there exists $i \in N$ such that $\overline{\mathrm{DA}}(\trianglerighteq_i', \trianglerighteq_{-i}) \triangleright_i \overline{\mathrm{DA}}(\trianglerighteq)$ for some $\trianglerighteq_i'$ such that $\trianglerighteq' \equiv (\trianglerighteq_i', \trianglerighteq_{-i})$ belongs to $\mathcal{D}_c$. Since $\overline{\mathrm{DA}}(\trianglerighteq_i', \trianglerighteq_{-i}) \neq \overline{\mathrm{DA}}(\trianglerighteq)$, we have two cases:

- *Case 1:* $\overline{\mathrm{DA}}_i(\trianglerighteq_i', \trianglerighteq_{-i}) \neq \overline{\mathrm{DA}}_i(\trianglerighteq)$. The definitions of $\overline{\mathrm{DA}}$ and $P_i(\trianglerighteq)$ ensure that

$$\mathrm{DA}_i(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq))\ P_i(\trianglerighteq)\ \mathrm{DA}_i(P(\trianglerighteq)).$$

  This implies that DA is not strategy-proof in $\mathcal{P}$, contradiction.

- *Case 2:* $\overline{\text{DA}}_i(\trianglerighteq_i', \trianglerighteq_{-i}) = \overline{\text{DA}}_i(\trianglerighteq) = s$. The definitions of $\overline{\text{DA}}$ and $\mathcal{D}_c$ ensure that

$$\begin{aligned} \text{DA}_i(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq)) &= \text{DA}_i(P(\trianglerighteq)) = s, \\ \text{DA}_s(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq)) &\neq \text{DA}_s(P(\trianglerighteq)). \end{aligned}$$

  This implies that DA is locally bossy, a contradiction to Theorem 1.

Therefore, the mechanism $\overline{\text{DA}} : \mathcal{D}_c \to \mathcal{M}$ is strategy-proof. □

Any stable mechanism $\Phi : \mathcal{P} \to \mathcal{M}$ induces a stable mechanism on $\mathcal{D}_c$ by the rule that associates each $\trianglerighteq \in \mathcal{D}_c$ with the matching $\Phi(P(\trianglerighteq))$. Evidently, many other stable mechanisms can be defined in $\mathcal{D}_c$ using the information that students reveal about how they rank their colleagues. Despite this multiplicity, the uniqueness result of Alcalde and Barberà (1994) can be extended to the preference domain $\mathcal{D}_c$.

Corollary 3. *On the preference domain $\mathcal{D}_c$ the mechanism $\overline{\text{DA}}$ is the only one that is stable and strategy-proof.*

Proof. It follows from Theorem 3 that $\overline{\text{DA}}$ is stable and strategy-proof on the preference domain $\mathcal{D}_c$. By contradiction, assume that there is a stable and strategy-proof mechanism $\Omega : \mathcal{D}_c \to \mathcal{M}$ such that $\Omega(\trianglerighteq) \neq \overline{\text{DA}}(\trianglerighteq)$ for some $\trianglerighteq \in \mathcal{D}_c$. Since $\mathcal{D}_c$ only includes school-lexicographic preferences, $\overline{\text{DA}}(\trianglerighteq)$ pairs each student to the most preferred alternative in $S \cup \{s_0\}$ that she can obtain in a stable outcome of $(S, N, \succ, q, \trianglerighteq)$.[14] Thus, $\overline{\text{DA}}_i(\trianglerighteq) \vartriangleright_i \Omega_i(\trianglerighteq)$ for some $i \in N$. In particular, $\overline{\text{DA}}_i(\trianglerighteq)$ is a school.

Let P_i' be the preferences defined on $S \cup \{s_0\}$ for which $\overline{\text{DA}}_i(\trianglerighteq)$ is the only acceptable school (i.e., $s_0 P_i' s$ for all school $s \neq \overline{\text{DA}}_i(\trianglerighteq)$). Fix $\trianglerighteq' = (\trianglerighteq_i', \trianglerighteq_{-i}) \in \mathcal{D}_c$ such that $P_i(\trianglerighteq') = P_i'$. Since the problems $(S, N, \succ, q, \trianglerighteq')$ and $(S, N, \succ, q, (P_i', P_{-i}(\trianglerighteq)))$ have the same stable matchings, and $\overline{\text{DA}}(\trianglerighteq)$ is stable under $(P_i', P_{-i}(\trianglerighteq))$, the definition of P_i' and the Rural Hospital Theorem (Roth, 1986)

[14] That is, there is no stable matching μ such that $\mu(i) P_i(\trianglerighteq) \overline{\text{DA}}_i(\trianglerighteq)$ for some $i \in N$.

imply that $\Omega_i(\trianglerighteq'_i, \trianglerighteq_{-i}) = \overline{\text{DA}}_i(\trianglerighteq)$. Therefore, $\Omega_i(\trianglerighteq'_i, \trianglerighteq_{-i}) \vartriangleright_i \Omega_i(\trianglerighteq)$, which contradicts the strategy-proofness of Ω. □

It is enough for the preferences of just one student to be school-lexicographic but not school-lexicographic over colleagues to prevent the existence of a stable and strategy-proof mechanism. The next example formalizes this claim by following the arguments made in the proof of Theorem 1 of Duque and Torres-Martínez (2023).

Example 2. Let $N = \{1, 2, 3, 4, 5\}$, $S = \{s_1, s_2, s_3, s_4\}$, $q_{s_1} = 2$, and $q_{s_k} = 1$ for all $k \neq 1$. Assume that schools' priorities satisfy the following conditions:

$$\succ_{s_1}: 4, 2, 1, 3, 5, \qquad \succ_{s_2}: 3, 2, \dots \qquad \succ_{s_3}: 1, 2, \dots \qquad \succ_{s_4}: 2, 5, \dots.$$

Let $\mathcal{D}_1$ be the preference domain in which the student 1 has school-lexicographic preferences and the other students have school-lexicographic preferences over colleagues. We claim that no stable mechanism defined in $\mathcal{D}_1$ is strategy-proof.

Consider a preference profile $\trianglerighteq = (\trianglerighteq_i)_{i \in N} \in \mathcal{D}_1$ such that:

$$P_1(\trianglerighteq) : s_3 \dots; \qquad P_2(\trianglerighteq) : s_2, s_1, \dots; \qquad P_3(\trianglerighteq) : s_1, s_2, \dots;$$

$$P_4(\trianglerighteq) : s_1, \dots; \qquad P_5(\trianglerighteq) : s_4, \dots.$$

Notice that $[N, S, \succ, q, P(\trianglerighteq)]$ has only two stable matchings:

$$\begin{aligned} \mu &= ((1, s_3), (2, s_1), (3, s_2), (4, s_1), (5, s_4)), \\ \eta &= ((1, s_3), (2, s_2), (3, s_1), (4, s_1), (5, s_4)). \end{aligned}$$

Since there are no restrictions on how the student 1 ranks μ and η, because she is assigned to the same school in both matchings and has school-lexicographic preferences, assume that she strictly prefers μ to η under $\trianglerighteq_1$.

It is not difficult to verify that μ and η are the only stable matchings of $[N, S, \succ, q, \trianglerighteq]$. Hence, if a mechanism $\Phi : \mathcal{D}_1 \to \mathcal{M}$ is stable, then $\Phi(\trianglerighteq) \in \{\mu, \eta\}$.

Suppose that $\Phi(\trianglerighteq) = \mu$. If $\trianglerighteq'_2$ is a school-lexicographic preference over colleagues such that $P'_2 \equiv P_2(\trianglerighteq'_2, \trianglerighteq_{-2})$ satisfies $P'_2 : s_2, s_4, \dots$, then η is the only stable matching of $[N, S, \succ, q, (\trianglerighteq_2', \trianglerighteq_{-2})]$. Hence, the student 2 has incentives to manipulate Φ, because $s_2 P_2(\trianglerighteq) s_1$ implies that $\Phi(\trianglerighteq_2', \trianglerighteq_{-2}) = \eta \rhd_2 \mu = \Phi(\trianglerighteq)$.

Suppose that $\Phi(\trianglerighteq) = \eta$. If $\trianglerighteq_1'$ is such that $P_1' \equiv P_1(\trianglerighteq_1', \trianglerighteq_{-1})$ satisfies $P_1' : s_1, s_3, \ldots$, then μ is the only stable matching of $[N, S, \succ, q, (\trianglerighteq_1', \trianglerighteq_{-1})]$. Hence, the student 1 has incentives to manipulate Φ, because $\Phi(\trianglerighteq_1', \trianglerighteq_{-1}) = \mu \rhd_1 \eta = \Phi(\trianglerighteq)$.

We conclude that no stable mechanism defined in $\mathcal{D}_1$ is strategy-proof. □

Given a school choice context $[N, S, \succ, q]$ and a domain $\mathcal{D}'$ such that $\mathcal{D}_c \subsetneq \mathcal{D}' \subseteq \mathcal{D}$, to ensure that no mechanism defined in $\mathcal{D}'$ is stable and strategy-proof it is crucial that $\succ$ has an Ergin-cycle.[15] Indeed, the mechanism $\overline{\text{DA}}$ is stable and strategy-proof in $\mathcal{D}$ whenever $\succ$ is acyclic (see Duque and Torres-Martínez, 2023).

6. Concluding remarks

In this paper, we have shown that the bossiness of the DA mechanism is limited. When DA is implemented, it is well known that a student can affect another student's assignment by changing her preferences without modifying her school. We have demonstrated that this holds only for students not assigned to the same school. In other words, under DA, a student cannot change her classmates without changing her own school. Additionally, for any strategy-proof mechanism, local non-bossiness guarantees that no coalition of students assigned to the same school can misrepresent their preferences to either improve their assignments or maintain their school while modifying their classmates. Since DA is not group strategy-proof, our result implies that a successful manipulating coalition of DA must include students from different schools.

Local non-bossiness is not only interesting in itself but also because of its application to school choice problems with externalities. In this framework, even when students prioritize their assigned school first and then consider the

[15] The priority profile of Example 2 has an Ergin-cycle characterized by $(s, s') = (s_1, s_2)$, $(i, j, k) = (2, 1, 3)$, $N_s = \{4\}$, and $N_{s'} = \emptyset$.

assignment of others, a stable and strategy-proof mechanism may not exist (Duque and Torres-Martínez, 2023). However, if we restrict preferences to be such that only the school and the classmates matter, it turns out that DA induces the only stable and strategy-proof mechanism. The crucial property behind this result is precisely its local non-bossiness.

Although we have focused on many-to-one matching problems with responsive preferences, it might be interesting to investigate the role of local non-bossiness in more general frameworks, as many-to-one matching models with contracts (Hatfield and Milgrom, 2005) or many-to-many matching problems (Echenique and Oviedo, 2006). These extensions are of interest due to their potential applications to more complex labor or educational markets, such as school choice with affirmative action or part-time work assignments (e.g., medical interns in hospitals or teachers in public schools). However, to guarantee that a mechanism Φ is locally non-bossy using our approach, the following three properties are crucial: (i) Φ must be a stable mechanism; (ii) Φ must assign all students to the most/least preferred school they can attend in a stable matching; and (iii) any school that does not fill its quota in some stable matching must enroll the same set of students in any other stable outcome. Therefore, any attempt to extend our results to other contexts will likely require restricting the preference domain to ensure that properties analogous to (i)-(iii) hold. This is a matter for future research.

APPENDIX A. LOCAL NON-BOSINESS AND ITS IMPLICATIONS

Lemma 1. *If $\Phi : \mathcal{P} \to \mathcal{M}$ is a locally non-bossy and strategy-proof mechanism, then it is locally group strategy-proof.*

Proof. Given $P \in \mathcal{P}$ and $s \in S \cup \{s_0\}$, suppose that there is a coalition $C = \{i_1, \ldots, i_r\}$ contained in $\Phi_s(P)$ such that $\Phi_i(P'_C, P_{-C}) R_i \Phi_i(P)$ for all $i \in C$ and for some $P'_C = (P'_j)_{j \in C}$. We will prove that $\Phi_i(P'_C, P_{-C}) = \Phi_i(P)$ for all $i \in C$.

For each $i \in \Phi_s(P)$, let $\widehat{P}_i$ be the preference relation that places $\Phi_i(P'_C, P_{-C})$ at the top and keeps the other schools in the order induced by

P_i. Suppose that $\Phi_{i_1}(P) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$. The strategy-proofness of Φ implies that $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$ and $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) \widehat{P}_{i_1} \Phi_{i_1}(P)$. If $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) \neq \Phi_{i_1}(P'_C, P_{-C})$, the definition of $\widehat{P}_{i_1}$ implies that $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) P_i \Phi_{i_1}(P)$. Hence, $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) P_{i_1} \Phi_{i_1}(P)$, which is not possible. As a consequence, $\Phi_{i_1}(P) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$ ensures that $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) = \Phi_{i_1}(P'_C, P_{-C})$, which in turn implies that $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(P'_C, P_{-C})$, a contradiction. We conclude that $\Phi_{i_1}(P) = \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$, and the local non-bossiness of Φ implies that $\Phi_i(P) = \Phi_i(\widehat{P}_{i_1}, P_{-i_1})$ for all $i \in \Phi_s(P)$. Repeating the argument for students $i_2, \ldots, i_r$ we obtain that $\Phi_i(P) = \Phi_i(\widehat{P}_C, P_{-C})$ for all $i \in \Phi_s(P)$.

Since $\Phi_{i_1}(P'_C, P_{-C})$ is the most preferred alternative under $\widehat{P}_{i_1}$, it follows that $\Phi_{i_1}(P'_C, P_{-C}) \widehat{R}_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C \setminus \{i_1\}}, P_{-C})$. If $\Phi_{i_1}(P'_C, P_{-C}) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C \setminus \{i_1\}}, P_{-C})$, then $\Phi_{i_1}(P'_C, P_{-C}) \widehat{P}_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C \setminus \{i_1\}}, P_{-C})$, which contradicts the strategy-proofness of Φ. Moreover, as Φ is locally non-bossy, $\Phi_i(P'_C, P_{-C}) = \Phi_i(\widehat{P}_{i_1}, P'_{C \setminus \{i_1\}}, P_{-C})$ for all $i \in \Phi_s(P)$. Repeating the argument for $i_2, \ldots, i_r$, we obtain that $\Phi_i(P'_C, P_{-C}) = \Phi_i(\widehat{P}_C, P_{-C})$ for all $i \in \Phi_s(P)$. Therefore, $\Phi_i(P'_C, P_{-C}) = \Phi_i(P)$ for every student $i \in C$. □

In Example 4 we show that the converse of Lemma 1 does not hold.

Afacan (2012) introduces the following extension of non-bossiness to group of students: a mechanism Φ is **group non-bossy** when for any $P \in \mathcal{P}$ and $C \subseteq N$, if there exists $P'_C \in \mathcal{L}^{|C|}$ such that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$, then $\Phi(P) = \Phi(P'_C, P_{-C})$.

We will consider a local version of this property: a mechanism Φ is **locally group non-bossy** when for any $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, and $C \subseteq \Phi_s(P)$, if there exists $P'_C \in \mathcal{L}^{|C|}$ such that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$, then $\Phi_s(P) = \Phi_s(P'_C, P_{-C})$.

Hence, local group non-bossiness ensures that no coalition of students assigned to the same school can change the assignment of some of their classmates by reporting different preferences, without changing their school.

Lemma 2. *If $\Phi : \mathcal{P} \to \mathcal{M}$ is a locally non-bossy and strategy-proof mechanism, then it is locally group non-bossy.*

Proof. Given $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, $C \subseteq \Phi_s(P)$, and $P'_C \in \mathcal{L}^{|C|}$, suppose that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$. Since $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ ensures that $\Phi_i(P'_C, P_{-C}) R_i \Phi_i(P)$, the arguments made in the proof of Lemma 1 guarantee that $\Phi_j(P) = \Phi_j(P'_C, P_{-C})$ for all $j \in \Phi_s(P)$. Hence, $\Phi_s(P) \subseteq \Phi_s(P'_C, P_{-C})$. Moreover, swapping the roles of P and (P'_C, P_{-C}), we obtain $\Phi_s(P'_C, P_{-C}) \subseteq \Phi_s(P)$. Therefore, $\Phi_s(P) = \Phi_s(P'_C, P_{-C})$. □

The converse of Lemma 2 does not hold. Indeed, consider the *Boston mechanism*, also known as the Immediate Acceptance mechanism. This mechanism runs similarly to DA, with the difference that at each step accepted students are definitively matched to the school. Although it is not strategy-proof (Abdulkadiroğlu and Sönmez, 2003), it is locally group non-bossy. Formally, denoting by $\mathcal{B} : \mathcal{P} \to \mathcal{M}$ this mechanism, suppose that $\mathcal{B}_i(P) = \mathcal{B}_i(P'_C, P_{-C}) = s$ for all student in a coalition $C \subseteq \mathcal{B}_s(P)$. Let $\widehat{P}_C = (\widehat{P}_i)_{i \in C}$ be such that s is the most preferred alternative under each $\widehat{P}_i$. It is not difficult to verify that $\mathcal{B}_s(P) = \mathcal{B}_s(\widehat{P}_C, P_{-C}) = \mathcal{B}_s(P'_C, P_{-C})$.

In what follows, we present examples that allow us to relate local non-bossiness and other properties.

Example 3. *A stable mechanism that is neither locally non-bossy nor strategy-proof.*

Let $N = \{1, 2, 3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2, 1)$, $\succ_{s_1}$: 1, 2, 3, and $\succ_{s_2}$: 3, 2, 1. Let $\overline{P} = (\overline{P}_1, \overline{P}_2, \overline{P}_3)$ be such that:

$$\overline{P}_1 : s_1, s_2, s_0, \qquad \overline{P}_2 : s_2, s_1, s_0, \qquad \overline{P}_3 : s_1, s_2, s_0.$$

Since the *school-optimal* stable mechanism in $[N, S, q, \succ, \overline{P}]$ is

$$\mathrm{DA}^S(\overline{P}) = ((1, s_1), (2, s_1), (3, s_2)),$$

it differs from $\mathrm{DA}(\overline{P}) = ((1, s_1), (2, s_2), (3, s_1))$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the stable mechanism such that $\Omega(P) = \mathrm{DA}(P)$ when $P \neq \overline{P}$, and $\Omega(\overline{P}) = \mathrm{DA}^S(\overline{P})$.

We claim that Ω is locally bossy. Let P_1 be such that $P_1 : s_2, s_1, s_0$. It is easy to see that $\Omega(P_1, \overline{P}_{-1}) = \text{DA}(P_1, \overline{P}_{-1}) = ((1, s_1), (2, s_2), (3, s_1))$. Since $\Omega(\overline{P}_1, \overline{P}_{-1}) = \text{DA}^S(\overline{P})$, when student 1's preferences change from P_1 to $\overline{P}_1$, she remains assigned to school s_1, but her classmates change as $\Omega_{s_1}(\overline{P}_1, \overline{P}_{-1}) = \{1, 2\} \neq \{1, 3\} = \Omega_{s_1}(P_1, \overline{P}_{-1})$.

Since DA is the only stable and strategy-proof mechanism (Alcalde and Barberà, 1994), it follows that Ω does not satisfy strategy-proofness. □

Since the Gale's Top Trading Cycles mechanism (Shapley and Scarf, 1974) is non-bossy and unstable, it follows from Example 3 that local non-bossiness and stability are independent properties.

Bando and Imamura (2016) introduce the following property: a mechanism Φ is **weak non-bossy** when for any $P \in \mathcal{P}$, $i \in N$, and $P'_i \in \mathcal{L}$, $\Phi_i(P) = \Phi_i(P'_i, P_{-i})$ implies that $\Phi_{s_0}(P) = \Phi_{s_0}(P'_i, P_{-i})$. Hence, when a mechanism is weak non-bossy, no student can modify the set of *unassigned students* without changing her own school.

Since DA satisfies weak non-bossiness (Bando and Imamura, 2016; Theorem 3), it is interesting to compare it with local non-bossiness.

Lemma 3. *Weak non-bossiness is independent of local non-bossiness and strategy-proofness.*

Proof. On the one hand, it is not difficult to verify that the mechanism described in Example 3 is weak non-bossy. However, this mechanism is neither locally non-bossy nor strategy-proof. On the other hand, if $N = \{1, 2, 3\}$, $S = \{s_1, s_2\}$, and $(q_{s_1}, q_{s_2}) = (2, 1)$, regardless of the schools' priority profile $(\succ_{s_1}, \succ_{s_2})$, the mechanism $\Omega : \mathcal{P} \to \mathcal{M}$ that is defined by

$$\Omega(P) = \begin{cases} ((1, s_1), (2, s_1), (3, s_2)), & \text{when } s_2 P_1 s_1, \\ ((1, s_1), (2, s_1), (3, s_0)), & \text{in another case,} \end{cases}$$

is locally non-bossy and strategy-proof but does not satisfy weak non-bossiness. □

In one-to-one matching problems, local non-bossiness only requires that student without school cannot modify the set of unassigned students without getting a place elsewhere. Thus, weak non-bossiness is stronger than local non-bossiness in this context.

Although any group strategy.proof mechanism is weak non-bossy, it follows from the proof of Lemma 3 that the combination of local non-bossiness and strategy-proofness does not implies on weak non-bossiness.

Group strategy-proofness is stronger than group non-bossiness (Afacan, 2012). However, the following example shows that the analogous result does not hold for the local versions of these concepts.

Example 4. *A locally group strategy-proof mechanism that is locally bossy and weak bossy.*

Let $N = \{1, 2, 3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2, 1)$, $\succ_{s_1}: 1, 2, 3$, and $\succ_{s_2}: 3, 1, 2$.

Let $\Omega : \mathcal{P} \to \mathcal{M}$ be such that $\Omega(P) = \text{DA}(P)$ unless the most preferred school of student 1 is s_1. In this case, let $\Omega_1(P) = s_1$ and $\Omega_i(P) = s_0$ for all $i \neq 1$. Notice that, since 1 has the highest priority at s_1, $\Omega_1(P) R_1 s_1$ for all $P \in \mathcal{P}$.

The mechanism Ω is strategy-proof, because DA satisfies this property and no one can prevent 1 to receive a seat in s_1 when it is her most preferred alternative. Moreover, Ω is locally group strategy-proof. Indeed, if this were not the case, there would exist $P, P' \in \mathcal{P}$ and two students in $\Omega_{s_1}(P) = \{i, j\}$ such that if they report (P'_i, P'_j) none of them would be worse off, and at least one of them strictly better off. Note that s_1 is not the most preferred school for student 1 under either P_1 (because in this case only one student is assigned to s_1) or P'_1 (because otherwise at least one of the students $\{i, j\}$ is not assigned under (P'_i, P'_j, P_k) and her situation worsens). Thus, Ω coincides with DA in the preference profiles P and (P'_i, P'_j, P_k). This contradicts the local group strategy-proofness of DA (see Theorem 1 and Lemma 1).

However, Ω is locally bossy. Consider the preference profile $P = (P_1, P_2, P_3)$:

$$P_1 : s_2, s_1, s_0, \qquad P_2 : s_1, s_2, s_0, \qquad P_3 : s_2, s_1, s_0.$$

We have that $\Omega(P) = ((1, s_1), (2, s_1), (3, s_2))$ and $\Omega_{s_1}(P) = \{1, 2\}$. If the student 1 changes her preference to P'_1 such that $P'_1 : s_1, s_2, s_0$, she remains assigned to school s_1 but is left without classmates because $\Omega_{s_1}(P'_1, P_2, P_3) = \{1\}$. This argument also shows that Ω is weak bossy, because $\Omega_{s_0}(P) = \varnothing$ and $\Omega_{s_0}(P'_1, P_2, P_3) = \{2, 3\}$. $\square$

Although the school-optimal stable mechanism is not strategy-proof, it is non-bossy (Afacan and Dur, 2017). Hence, the Example 4 implies that local non-bossiness and strategy-proofness are independent properties.

The last examples show that local group non-bossiness is stronger than local non-bossiness (Example 5), and than local group strategy-proofness is stronger that strategy-proofness (Example 6).

Example 5. *A locally non-bossy mechanism that is not locally group non-bossy.*

Let $N = \{1, 2, 3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (3, 1)$, $\succ_{s_1}: 1, 2, 3$, and $\succ_{s_2}: 3, 2, 1$.

Denote by $top(P_i)$ the most preferred alternative of student i when her preferences are $P_i \in \mathcal{L}$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the mechanism such that:

$$\Omega(P) = \begin{cases} ((1, s_1), (2, s_1), (3, s_2)), & \text{when } top(P_1) = top(P_2) = s_1; \\ ((1, s_1), (2, s_1), (3, s_1)), & \text{when } top(P_1) = top(P_2) = s_2; \\ ((1, s_0), (2, s_0), (3, s_0)), & \text{in another case.} \end{cases}$$

It is not difficult to verify that Ω is locally non-bossy: the only two students who might change their partners without changing schools are 1 and 2. However, if only one of them changes her preferences, she will be assigned to s_0 (if she was originally assigned to s_1 or s_2), or will not change anyone's assigned school otherwise.

Moreover, Ω is locally group bossy. Let $P = (P_1, P_2, P_3)$ be a preference profile such that $top(P_1) = top(P_2) = s_1$ and consider

$P_1', P_2' \in \mathcal{L}$ such that $top(P_1') = top(P_2') = s_2$. Notice that, although $\Omega_i(P) = \Omega_i(P_1', P_2', P_3) = s_1$ for each student $i \in \{1,2\}$, we have that $\Omega_{s_1}(P) = \{1,2\} \neq \{1,2,3\} = \Omega_{s_1}(P_1', P_2', P_3)$. □

Example 6. *A strategy-proof mechanism that is not locally group strategy-proof.*

Let $N = \{1,2,3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2,2)$, $\succ_{s_1}$: 1,2,3, and $\succ_{s_2}$: 3,2,1.

Denote by $top_k(P_i)$ the k-th most preferred alternative of student i under $P_i \in \mathcal{L}$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the mechanism such that:

$$\Omega(P) = \begin{cases} ((1, top_1(P_1)), (2, s_1), (3, s_2)), & \text{when } top_2(P_1) = s_2; \\ ((1, top_1(P_1)), (2, s_2), (3, s_1)), & \text{in another case.} \end{cases}$$

It is easy to verify that Ω is strategy-proof. We claim that Ω is not locally group strategy-proof. Let $P = (P_1, P_2, P_3)$ be a preference profile such that $P_1 : s_1, s_2, s_0$ and $P_2 : s_2, s_1, s_0$. Let $P_1', P_2' \in \mathcal{L}$ be such that $P_1' : s_1, s_0, s_2$ and $P_2' = P_2$. It follows that the coalition of classmates $\{1,2\} \in \Omega_{s_1}(P)$ can manipulate the mechanism Ω at P, because $\Omega_1(P) = s_1 = \Omega_1(P_1', P_2', P_3)$ and $\Omega_2(P_1', P_2', P_3) = s_2 P_2 s_1 = \Omega_2(P)$. □

The following diagram summarizes the causal relationships between local non-bossiness and other incentive properties:

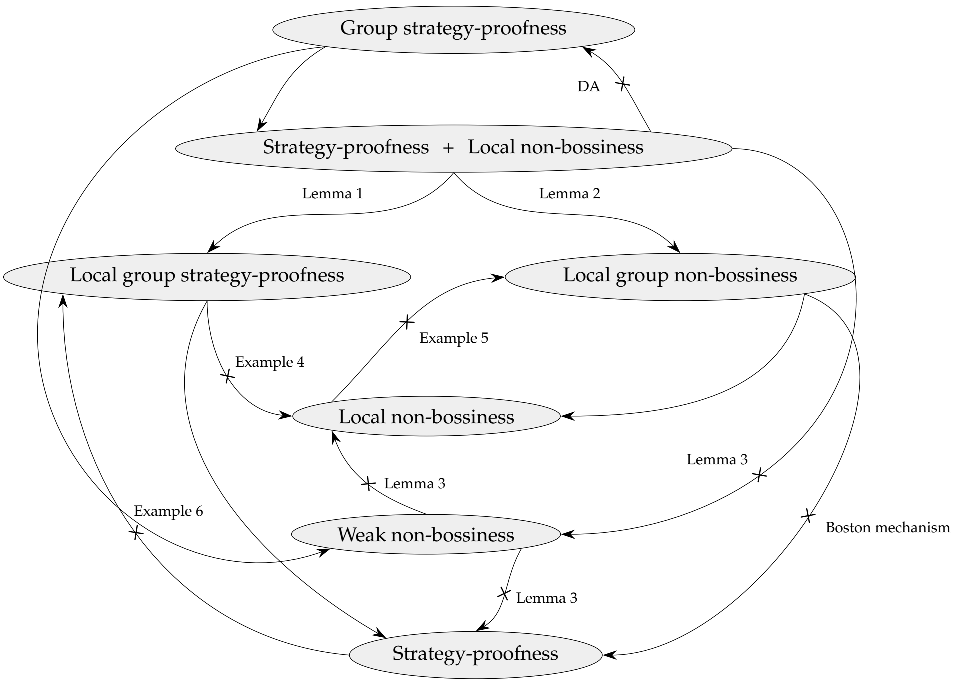

FIGURE 1. On local non-bossiness and other incentive properties.

With the information provided in Figure 1, it can be easily inferred that all absent causal relationships are not satisfied.

APPENDIX B. OMITTED PROOFS

Given a student $1 \in N$ and preference profiles $P, P' \in \mathcal{P}$, let $\mu = \text{DA}(P_1, P_{-1})$ and $\mu' = \text{DA}(P_1', P_{-1})$. In the following results we characterize some properties of μ and μ'.

Lemma 4. *Suppose that $\mu(1) = \mu'(1) = \bar{s}$. If there exists $i \in N$ such that $s = \mu(i)$ and $s' = \mu'(i)$ are different, then $s \neq s_0$, $s' \neq s_0$, and both schools fill their places in μ and μ'.*

Proof. Let $\widehat{\mu}$ be the matching induced by μ when student 1 and one seat in $\bar{s}$ are eliminated (w.l.o.g., assume that $q_{s_0} = |N|$). Define $\widehat{\mu}'$ analogously. Notice that $\widehat{\mu}$ and $\widehat{\mu}'$ are stable matchings in the school choice problem $(N \setminus \{1\}, S, (\succ_r)_{r\in S}, (\widehat{q}_r)_{r\in S}, P_{-1})$ where $\widehat{q}_{\bar{s}} = q_{\bar{s}} - 1$ and $\widehat{q}_r = q_r$ for $r \neq \bar{s}$. Since $s = \widehat{\mu}(i)$ and $s' = \widehat{\mu}'(i)$ are different, the Rural Hospital Theorem (Roth, 1986) implies that s and s' are schools and fill their places in both $\widehat{\mu}$ and $\widehat{\mu}'$.[16] Therefore, s and s' satisfy the same properties in μ and μ'. □

Lemma 5. *If $\mu'(i)P_i\mu(i)$, then there exists a cycle $(i_1, \ldots .i_r)$ that includes i and satisfies:*

(i) $\mu'(i_l) = \mu(i_{l+1})$ *for all* $l \in \{1, \ldots, r\}$ *[modulo r].*
(ii) $\mu'(i_l)P_{i_l}\mu(i_l)$ *for all* $l \in \{1, \ldots, r\}$.

Proof. Let $G^* = (V^*, E^*)$ be a *multigraph* in which $V^* = S$ and there is a directed edge between s and s' *for each* student k such that $\mu(k) = s$ and $\mu'(k) = s'$, with $s \neq s'$. Since Lemma 4 ensures that $|\mu(s)| = q_s$ and $|\mu'(s')| = q_{s'}$ for any s and s' such that $[s, s'] \in E^*$, the in-degree and the out-degree of each node in V^* coincide. Moreover, as schools $a = \mu(i)$ and $a' = \mu'(i)$ are different, there is at least one edge between a and a'.

As a consequence, there is a cycle of schools $(s_1, \ldots, s_r)$ in G^* such that $(s_1, s_2) = (a, a')$. Indeed, we can form this cycle departing from node a in direction to node a' and moving around the graph, because after enter a node $s_j \neq s_1$ from an edge, we are always able to exit from another edge. Since V^* is a finite set, we will eventually return to s_1. The construction of E^* allow us to associated to $(s_1, \ldots, s_r)$ a group of students $i_1, \ldots, i_r$ such that $i_1 = i$, $\mu(i_l) = s_l$ for all $l \in \{1, \ldots, r\}$, and $\mu'(i_l) = s_{l+1} = \mu(i_{l+1})$ for all $l \in \{1, \ldots, r\}$ [modulo r]. In particular, although $s_1, \ldots, s_r$ are not necessarily different, we can assume that the students $i_1, \ldots, i_r$ are.

[16] In many-to-one matching problems, not only are the unassigned students the same in all stable matchings, but each school that does not fill its places enrolls the same group of students (Roth, 1986).

Since $\mu'(i_l) = s_{l+1}$, when $\mu'(i_l) P_{i_l} \mu(i_l)$ the stability of μ ensures that $j \succ_{s_{l+1}} i_l$ for all $j \in \mu(s_{l+1})$. Hence, the stability of μ' implies that any edge between s_{l+1} and s_{l+2} is associated to a student that improves we she moves from μ to μ'. In particular, $\mu'(i_{l+1}) P_{i_{l+1}} \mu(i_{l+1})$. Therefore, $\mu'(i_1) P_{i_1} \mu(i_1)$ ensures that $\mu'(i_l) P_{i_l} \mu(i_l)$ for all $l \in \{1, \dots, r\}$.

We conclude that $(i_1, \dots, i_r)$ satisfies properties (i) and (ii). □

Lemma 6. *Suppose that* $|N| = \sum_{s \in S} q_s$, $P \in \mathcal{P}^*$, *and* $(P'_1, P_{-1}) \in \mathcal{P}^*$. *If* $\mu(1) \neq \mu'(1)$, *then* $\text{Coll}_1(P) \cap \text{Coll}_1(P'_1, P_{-1}) = \emptyset$.

Proof. Since $|N| = \sum_{s \in S} q_s$ and everyone considers all schools admissible under the preference profiles P and (P_1, P_{-1}), it follows that $\bar{s} \equiv \mu(1)$ and $\hat{s} \equiv \mu'(1)$ are two different schools. By contradiction, suppose that there exists $i_1 \in N \setminus \{1\}$ such that $\mu(i_1) = \bar{s}$ and $\mu'(i_1) = \hat{s}$. Without loss of generality, assume that $\hat{s} P_{i_1} \bar{s}$ (in other case, exchange the roles of P and (P'_1, P_{-1})).

In the proof of Theorem 1, the fact that $\mu(1) = \mu'(1)$ is only used to guarantee that all students who have different assignments in μ and μ' are assigned to schools and these schools fill their quota (see Lemma 4). In the current context, although $\mu(1) \neq \mu'(1)$, the same properties hold because DA is a stable mechanism, $|N| = \sum_{s \in S} q_s$, and preference profiles belong to $\mathcal{P}^*$.

Thus, after assuming the existence of $i_1 \neq 1$ such that $\mu(i_1) = \bar{s}$, $\mu'(i_1) = \hat{s}$, and $\hat{s} P_{i_1} \bar{s}$, the same arguments made in the proof of Theorem 1 ensure that $\mu(\bar{s}) \setminus \{1\} = \mu'(\bar{s}) \setminus \{1\}$. Since $i_1 \in \mu(\bar{s}) \setminus \{1\}$, we conclude that $\bar{s} = \hat{s}$. A contradiction. □

Proof of Theorem 2. Given a priority profile $\succ$, the stability of $\text{DA}^{\succ}$ ensures that it is individually rational and weakly non-wasteful. $\text{DA}^{\succ}$ is also population-monotonic (Crawford, 1991), strategy-proof (Dubins and Freedman, 1981; Roth, 1982), and weakly WrARP because under deferred acceptance each school chooses the students according to its priority order. The weak local non-bossiness of $\text{DA}^{\succ}$ follows from Theorem 1.

Suppose that $\Phi : \mathcal{N} \times \mathcal{P} \to \mathcal{M}^*$ satisfies individual rationality (IR), weak non-wastefulness (WNW), population-monotonicity (PM), strategy-proofness (SP), weak WrARP (w-WrARP), and weak local non-bossiness (WLNB).

Given $s \in S$, let P^s be a preference relation such that s is the only admissible school. Denote by $\overline{P} = (\overline{P}_i)_{i \in \overline{N}}$ a preference profile such that $\overline{P}_i = P^s$ for every $i \in \overline{N}$. It follows from WNW that the mapping that associates to each $N \in \mathcal{N}$ the set $\Phi_s(N, \overline{P})$ is a choice function.[17] Moreover, WNW, PN, w-WrARP, and the Theorem 2 of Chambers and Yenmez (2018) guarantee that there exists a priority order $\succ_s$ defined on $\overline{N}$ such that the following property holds:

(A) For each $N \in \mathcal{N}$, if $i \in N \setminus \Phi_s(N, \overline{P})$, then $j \succ_s i$ for all $j \in \Phi_s(N, \overline{P})$.

Let $\succ = (\succ_s)_{s \in S}$. We will use the property (A) to show that $\Phi = \text{DA}^{\succ}$.

By contradiction, assume that there is $N^* \in \mathcal{N}$ such that $\Phi(N^*, \cdot) \neq \text{DA}^{\succ}(N^*, \cdot)$. For each $(N, P) \in \mathcal{N} \times \mathcal{P}$, let $Z(N, P) = |\{i \in N : |A(P_i)| \leq 1\}|$ be the number of students in N who find at most one school admissible. Let $P^* \in \mathcal{P}$ be such that $\Phi(N^*, P^*) \neq \text{DA}^{\succ}(N^*, P^*)$ and $Z(N^*, P^*) \geq Z(N^*, P)$ for all $P \in \mathcal{P}$ satisfying $\Phi(N^*, P) \neq \text{DA}^{\succ}(N^*, P)$. Given $i \in N^*$, we claim that $\Phi_i(N^*, P^*) \neq \text{DA}^{\succ}_i(N^*, P^*)$ implies that $|A(P^*_i)| = 1$. There are two cases to analyze:

- *Case 1:* $\Phi_i(N^*, P^*) \, P^*_i \, \text{DA}^{\succ}_i(N^*, P^*)$. Since Φ and $\text{DA}^{\succ}$ satisfy IR, $s := \Phi_i(N^*, P^*)$ belongs to S. Hence, $|A(P^*_i)| \geq 1$. IR and SP imply that $\Phi_i(N^*, (P^s, P^*_{-i})) = s$ and $\text{DA}^{\succ}_i(N^*, (P^s, P^*_{-i})) = s_0$. As a consequence, if $|A(P^*_i)| > 1$, we obtain that $Z(N^*, (P^s, P^*_{-i})) > Z(N^*, P^*)$ and $\Phi(N^*, (P^s, P^*_{-i})) \neq \text{DA}^{\succ}(N^*, (P^s, P^*_{-i}))$, which contradicts the definition of P^*. Therefore, $|A(P^*_i)| = 1$.
- *Case 2:* $\text{DA}^{\succ}_i(N^*, P^*) \, P^*_i \, \Phi_i(N^*, P^*)$. The result follows from the same arguments applied in the previous case but swapping the roles of Φ and $\text{DA}^{\succ}$.

Let $i \in N^*$ be such that $\Phi_i(N^*, P^*) \neq \text{DA}^{\succ}_i(N^*, P^*)$. Since $|A(P^*_i)| = 1$, it follows from IR that there exists $s \in S$ such that either $s =$

[17] A *choice function* is a mapping $C : \mathcal{N} \to \mathcal{N}$ such that, for each $N \in \mathcal{N}$, $C(N)$ is a non-empty subset of N.

$DA_i^{\succ}(N^*,P^*)\, P_i^*\, \Phi_i(N^*,P^*) = s_0$ or $s = \Phi_i(N^*,P^*)\, P_i^*\, DA_i^{\succ}(N^*,P^*) = s_0$. Furthermore, without loss of generality, IR and SP allows us to assume that $P_i^* = P^s$.

When $s = DA_i^{\succ}(N^*,P^*)\, P_i^*\, \Phi_i(N^*,P^*) = s_0$, WNW implies that $|\Phi_s(N^*,P^*)| = q_s$. Let $\Phi_s(N^*,P^*) = \{i_1,\ldots,i_{q_s}\}$, where $i_1 \succ_s \cdots \succ_s i_{q_s}$. Since IR and SP ensure that $\Phi_{i_1}(N^*,(P^s,P^*_{-i_1})) = s$, it follows from WLNB that $\Phi_s(N^*,(P^s,P^*_{-i_1})) = \Phi_s(N^*,P^*)$. Applying this argument to $i_2,\ldots,i_{q_s}$, we conclude that there is no loss of generality in assuming that $P_j^* = P^s$ for all $j \in \Phi_s(N^*,P^*)$.[18] Let $N = \{i_1,\ldots,i_{q_s},i\}$. Since $P_i^* = P^s$, $N \subseteq N^*$, and $DA_i^{\succ}(N^*,P^*) = s$, it follows from PM that $DA_i^{\succ}(N,P^*) = s$. Moreover, the fact that $P_N^* = (P^s,\ldots,P^s)$ and the definition of $DA^{\succ}$ guarantee that $DA_{i_{q_s}}^{\succ}(N,P^*) = s_0$ and $i \succ_s i_{q_s}$. On the other hand, since $N \subseteq N^*$ and $\Phi_s(N^*,P^*) = N \setminus \{i\}$, PM implies that $\Phi_s(N,P^*) = N \setminus \{i\}$.[19] Hence, $\Phi_i(N,P^*) = s_0$ and the property (A) guarantees that $i_{q_s} \succ_s i$. A contradiction.

When $s = \Phi_i(N^*,P^*)\, P_i^*\, DA_i^{\succ}(N^*,P^*) = s_0$, we can obtain a contradiction applying the arguments of the previous paragraph but swapping the roles of Φ and $DA^{\succ}$. □

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[18] To adapt Ehlers and Klaus (2016, Theorem 1) to many-to-one matching problems, it is crucial to show that w.l.o.g. it can be assumed that $P_j^* = P^s$ for all $j \in \Phi_s(N^*,P^*)$ whenever $\Phi_s(N^*,P^*) \neq DA_s^{\succ}(N^*,P^*)$. The same arguments of these authors allow us to show that this property holds for each $j \in \Phi_s(N^*,P^*) \setminus DA_s^{\succ}(N^*,P^*)$ when Φ satisfies IR, WNW, PM, and SP. Hence, the weak local non-bossiness of Φ allows us to ensure this property for the students in $\Phi_s(N^*,P^*) \cap DA_s^{\succ}(N^*,P^*)$, if any. Notice that, when $q_s = 1$, $\Phi_s(N^*,P^*) \neq DA_s^{\succ}(N^*,P^*)$ implies that $\Phi_s(N^*,P^*) \cap DA_s^{\succ}(N^*,P^*)$ is an empty set.

[19] Remember that $P_N^* = (P^s,\ldots,P^s) = \overline{P}_N$ and $\Phi(N,P^*)$ is independent of the preferences $P^*_{\overline{N}\setminus N}$.

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