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ORIGINAL PAPER

Preservice mathematics teachers' noticing in AI-based simulations: transitions among attending, interpreting, and shaping

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Abstract

Teacher noticing is a key component of teacher competence, referring to how teachers attend to, interpret, and shape classroom interactions to gain access to students' thinking. However, less is known about how these dimensions are coordinated during ongoing interaction. This study addresses this gap by conceptualising noticing as an iterative and interactional process and examining transitions among noticing dimensions within AI-based simulations. Seventeen preservice mathematics teachers participated in a three-week intervention involving text-based interactions with AI-simulated students and mentor feedback. Data were analysed using qualitative coding and Epistemic Network Analysis. Findings show that noticing was organised through recurring transitions among attending, interpreting, and shaping. Strong connections between interpreting and shaping suggest that PTs' interpretations frequently informed subsequent inquiry moves that generated new opportunities for noticing students' thinking. Differences across weeks further indicate the situated and context-sensitive organisation of noticing within AI-based interactions. The study contributes to demonstrating how AI-based simulations can support and make visible the iterative coordination of noticing dimensions in teacher education.

Keywords Teacher noticing · Artificial intelligence · Chatbot · Teacher education · Mathematics education

1 Introduction

Over the past two decades, teacher noticing has become a well-established and widely investigated construct in mathematics education (Jacobs et al., 2010; van Es & Sherin, 2002, 2021), as it has been linked to high-quality teaching (Jacobs & Spangler, 2017) and student learning (Kaiser et al., 2017). A substantial body of research has shown that expert teachers demonstrate more sophisticated noticing practices than novices, particularly in their selective attention to critical events and their ability to make sense of students' reasoning (Berliner, 1994; Jacobs et al., 2010). However, preservice teachers (PTs) often struggle to prioritise students' mathematical thinking and require structured opportunities to develop this capacity (Cai et al., 2022; Prediger & Zindel, 2017).

Teacher noticing is commonly conceptualised as comprising multiple interrelated dimensions, most prominently attending, interpreting, and responding (Jacobs et al., 2010). Recent perspectives (van Es & Sherin, 2026) have introduced shaping to emphasise teachers' active role in creating interactions to gain insight into students' thinking. While these dimensions are often presented as analytically distinct, noticing is increasingly understood as dynamic, situated, and interactional (van Es & Sherin, 2021). Rather than unfolding in a linear sequence, noticing processes are conceptualised as mutually influential and recursively related, shaped through teachers' engagement with students and instructional contexts (Thomas et al., 2023; Scheiner, 2026). Despite this theoretical shift, empirical research has struggled to capture how these dimensions are coordinated in practice. Much of the existing literature has examined noticing through isolated components or aggregated measures, providing limited insight into how noticing unfolds as a coordinated process over time (Kersting et al., 2026). As a result, there remains a need for analytic approaches that make visible how teachers move among noticing dimensions during interaction (Thomas et al., 2023). Addressing this gap requires a process-oriented perspective that cap-

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tures the dynamic relationships among attending, interpreting, and shaping as they emerge in practice.

Simulation-based environments have been identified as a promising approach for supporting the development of teacher noticing, particularly by providing structured and low-risk opportunities to engage with student thinking (Castro Superfine et al., 2026; Howell & Mikeska, 2021). Such environments are especially valuable for PTs, for whom limited classroom experience makes real-time noticing both demanding and challenging (Keskin et al., 2024). Building on these developments, AI-based simulations have increasingly been recognised as a promising direction in teacher education (Bernardi et al., 2025; Zhuang & Zhang, 2025). In these environments, AI can take on the role of a virtual student, allowing PTs to rehearse questioning practices (Lee & Yeo, 2022; Son et al., 2024; Zhuang & Zhang, 2025) and engage in diagnostic noticing of student thinking (Bastian et al., 2025; Galiç et al., 2025). This approach aligns with the inherently interactive nature of noticing (van Es & Sherin, 2021). However, relatively little is known about how noticing unfolds within such environments and how these contexts shape the coordination of noticing dimensions.

This study addresses these gaps by conceptualising teacher noticing as a dynamic, interactional process and examining how PTs transition among attending, interpreting, and shaping within AI-based simulations. In this study, we conceptualise AI not as a replacement for human expertise but as a supportive tool for enhancing PTs' professional practice (Çukurova, 2025). AI chatbots are positioned as socio-interactive learning partners rather than mere information providers (Wu et al., 2025). We refer to PTs' interactions with large language models (LLMs) as AI-based simulations, in which they engage in conversations with virtual students and receive feedback from a virtual mentor.

Our study builds on three core insights from the literature. First, expert teachers demonstrate more sophisticated noticing than novices, while PTs require targeted opportunities to develop this capacity (Berliner, 1994; Cai et al., 2022; Jacobs et al., 2010). Second, noticing students' thinking is central to effective teaching, yet PTs often struggle to prioritise students' mathematical reasoning (Prediger & Zindel, 2017), although such focus can develop with appropriate practice (van Es & Sherin, 2021). Third, although noticing is commonly described in terms of multiple dimensions, there is limited empirical work examining how these dimensions interact and how teachers move between them during ongoing practice. Emerging perspectives emphasise reflexive and recursive relationships among noticing dimensions (Coles & Helliwell, 2025; Thomas et al., 2023). However, how transitions among these dimensions emerge and evolve within interactive contexts remains largely unexplored. In the present study, we focus specifically on transitions in PTs' noticing practices within AI-based simulations. The simulations were designed to foreground one-to-one teacher–student interaction, enabling focused engagement with student thinking. Accordingly, the study addresses the following research question:

> How do preservice mathematics teachers transition among attending, interpreting, and shaping within AI-based simulations?

2 Literature review

2.1 Teacher noticing in mathematics education

Teacher noticing has emerged as a central construct in mathematics education research, reflecting a broader shift towards understanding teaching as a responsive, student-centred practice. Rooted in the notion of professional vision (Goodwin, 1994), teacher noticing refers to the ways in which teachers attend to, interpret, and act upon classroom events. Early conceptualisations framed teacher noticing as a set of interrelated cognitive processes. van Es and Sherin (2008) conceptualised noticing as an integrated process of selective attention and knowledge-based reasoning, emphasising that teachers must identify salient classroom features and make sense of them in relation to broader pedagogical goals. These perspectives align with Mason's (2002) notion of noticing as a disciplined practice underpinning professional growth. Similarly, Jacobs and her colleagues (2010) defined professional noticing as comprising attending to students' strategies, interpreting their understanding, and deciding how to respond instructionally.

More recent scholarship extends these perspectives by foregrounding the situated, interactive, and dynamic nature of noticing. van Es and Sherin (2021) introduced shaping as a further dimension, highlighting how teachers actively construct opportunities to access student thinking. Teachers engage in practices such as eliciting details and leveraging classroom structures to deepen their interpretations (van Es & Sherin, 2026). In this view, noticing is not merely reactive but involves actively organising the instructional environment to make student thinking visible. This shift reflects a broader reconceptualisation of noticing as an iterative and relational practice. For instance, Stockero et al. (2025) conceptualise teacher noticing as an iterative, dynamic, and ongoing process. Similarly, enactivist perspectives conceptualise noticing as embodied and socially mediated, shaped by affective and contextual factors (Coles & Helliwell, 2025). These perspectives collectively challenge earlier views of noticing as a purely cognitive skill and instead position it as a complex, context-sensitive practice embedded in interaction. At the same time, research has highlighted that noticing is shaped by teachers' interpretive frames and orientations. Studies (e.g., Scheiner, 2026) indicate that deficit-oriented framings may limit teachers' ability to recognise the potential in students' ideas, whereas strength-based perspectives support more equitable instructional responses.

Moreover, noticing is increasingly understood as domain- and content-sensitive (Copur-Gencturk & Rodrigues, 2021). Findings suggest that what teachers notice is closely tied to the specific mathematical content and instructional context. Building on this, recent research has begun to emphasise content-specific noticing, arguing that teachers' ability to attend to and interpret student thinking is deeply intertwined with their understanding of mathematical ideas. In the next sub-section, we present noticing of secondary school students' understanding of equal sign since we focus on the equal sign as a context in this study.

2.2 Domain-specific noticing in mathematics education

Domain-specific noticing in mathematics education refers to teachers' selective attention to, and interpretation of, classroom events that are directly tied to mathematical content and students' conceptual thinking. It demands deep subject-matter knowledge and the ability to link observations with disciplinary representations (Sherin et al., 2011). Large-scale evidence shows that while teachers frequently notice content-specific aspects, relatively few are able to identify the critical mathematical issues underlying students' struggles (Copur-Gencturk & Rodrigues, 2021). Moreover, expertise in this form of noticing is not automatically acquired through experience; rather, it develops through deliberate engagement with cognitively demanding tasks and targeted professional development (Prediger et al., 2016).

Teachers' analyses of representational use differ across mathematical domains, though a general competence dimension can still be inferred (Friesen & Kuntze, 2021; Dreher & Kuntze, 2015). Design research on professional development has demonstrated that fostering teachers' noticing of students' mathematical potentials requires both fine-grained attention to the content and sustained opportunities for reflection on classroom practice (Prediger et al., 2016). From this perspective, we adopt domain-specific noticing and focus on the equal sign concept in this study as a situated, knowledge-dependent. In this study, we conceptualise teacher noticing as domain-specific and investigate PTs' noticing in the context of the equal sign.

Early knowledge of the equal sign among primary school students predicts later algebraic performance (Matthews & Fuchs, 2020). Students who understand the equal sign as an indicator of equivalence demonstrate stronger performance in solving algebraic equations and employ more sophisticated strategies in tasks involving equivalent expressions (Knuth et al., 2008). In contrast, interpreting the equal sign as merely a prompt to compute constitutes a significant barrier to algebra learning (Byrd et al., 2015). Despite its central role, students across educational levels experience persistent difficulties in understanding the equal sign as a symbol of equivalence (Matthews et al., 2012; Parodi et al., 2017, 2020). Moreover, teachers often lack awareness of these challenges and encounter difficulties in anticipating instructional goals, solution strategies, or students' errors in tasks involving the equal sign (Parodi et al., 2024; Parodi & Ochoviet, 2025).

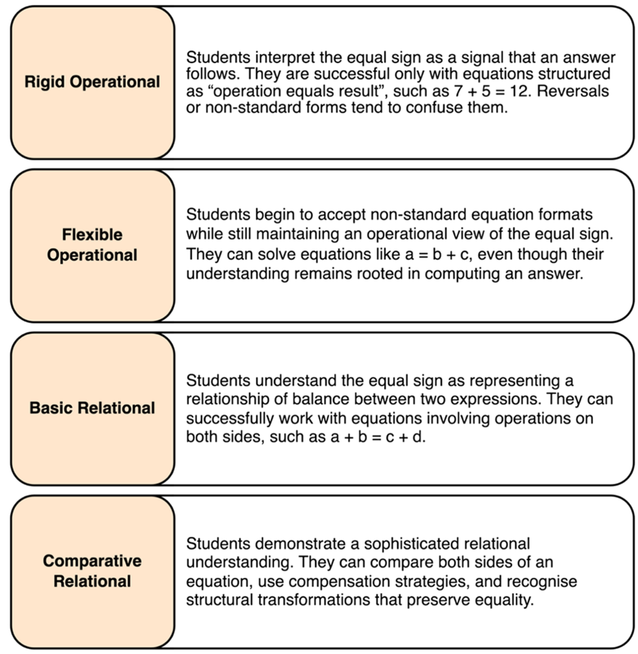


Fig. 1 Knowledge of the Equal Sign (Matthews et al., 2012, p. 320)

Given its foundational role in algebraic reasoning, particularly at the secondary level, understanding the equal sign as equivalence remains critical (Matthews et al., 2012; Parodi et al., 2017, 2020). Building on this perspective, this study adopts a domain-specific noticing approach to examine pre-service teachers' noticing of students' understanding of the equal sign (Matthews et al., 2012, p. 320), framing it as equivalence to guide the analytic framework (Fig. 1).

Although the conceptual understanding of the equal sign has been extensively examined from the perspective of students' learning, studies focusing on how teachers notice students' mathematical thinking in this context remain limited (Parodi et al., 2024). Therefore, investigating teacher noticing concerning the equal sign is essential for gaining a more comprehensive understanding of the current state of teacher noticing. In this study, we focus on teacher noticing in the context of the equal sign through AI-based simulations.

2.3 The use of GenAI in teacher education

Generative AI (GenAI) enables interactive, language-rich simulations in which PTs engage with representations of student thinking. Unlike earlier simulation tools with fixed or closed response options, GenAI can simulate both mathematically coherent reasoning and misconception-laden thinking, offering PTs opportunities to engage in practices such as eliciting, interpreting, and responding to student ideas in dynamic ways (Galiç et al., 2025; Zhuang & Zhang, 2025). The dual functionality, acting as a virtual student and or mentor, enables PTs to rehearse instructional moves while also receiving feedback or prompts that guide reflection on their pedagogical decisions (Son et al., 2024). Such environments align with practice-based approaches, allowing PTs to decompose complex teaching into manageable components and rehearse them in low-stakes contexts (Lee & Yeo, 2022). Moreover, GenAI-mediated dialogue has been shown to contribute to teachers' professional knowledge when teachers actively engage in critical evaluation and adaptation of AI-generated suggestions (Segal & Klemer, 2025).

A growing body of research highlights several key affordances of GenAI in teacher education. First, AI-based simulations provide scalable and repeatable practice opportunities, enabling PTs to engage in iterative cycles of questioning, interpreting, and responding without the constraints of classroom access. These environments support the development of responsive teaching practices by allowing PTs to experiment with different questioning strategies and receive immediate feedback (Son et al., 2024). Second, GenAI exposes PTs to a broader range of student thinking, including uncommon misconceptions that may not frequently arise in field placements. This exposure is particularly valuable for developing pedagogical content knowledge and preparing teachers to respond flexibly to diverse learner needs (Zhuang & Zhang, 2025). Third, the dialogic nature of LLMs interaction supports reflective practice by positioning AI as a conversational partner that can prompt deeper consideration of instructional decisions and alternatives (Segal & Klemer, 2025). Despite these affordances, important limitations persist. One major concern is the lack of authenticity in AI-generated interactions. While LLMs can simulate student reasoning, their discourse may not accurately reflect age-appropriate language, affective responses, or the social dynamics of real classrooms. This mismatch can make it difficult for PTs to transfer strategies from simulation to practice (Zhuang & Zhang, 2025). Additionally, AI-generated responses may be overly verbose, mathematically inaccurate, or epistemically misleading, requiring scrutiny by users (Bernardi et al., 2025). Broader reviews also highlight risks such as bias, hallucination, and over-reliance on GenAI, all of which underscore the need for human oversight and critical engagement (Pepin et al., 2025).

AI-based tools offer a promising alternative by providing accessible, scalable, and interactive environments for teacher noticing (Galiç et al., 2025). LLMs can direct PTs' attention to key features of student thinking, support interpretation by prompting them to refine their understanding, and scaffold responding by encouraging instructional moves (Bastian et al., 2025). Furthermore, PTs' noticing expertise is closely related to the quality of their instructional decisions, highlighting the importance of structured opportunities to practice noticing in conjunction with teaching moves (Son, 2025). On the other hand, AI-generated representations of student thinking may shape what PTs attend to, potentially narrowing their focus (Galiç et al., 2025). This suggests that without careful design and facilitation, GenAI may reinforce partial or unbalanced forms of noticing. GenAI extends existing representations of practice by offering interactive, adaptive, and dialogic simulations that can support the development of teacher noticing. However, its effectiveness depends on critical use, pedagogical framing, and integration into structured learning environments.

3 Theoretical framework of this study

In this study, we adopt the Teacher Noticing of Rationality Framework (TNRF; Galiç et al., in press) as our theoretical framework. The TNRF builds on the expanded learn to notice framework (van Es & Sherin, 2021), which conceptualises teacher noticing as comprising three interrelated dimensions: attending, interpreting, and shaping. This perspective positions noticing as an active and interactional practice in which teachers selectively attend to classroom events, interpret their significance, and construct interactions to gain access to students' thinking. TNRF (Galiç et al., in press) extends van Es and Sherin's (2021) framework by specifying the object of noticing more precisely as students' mathematical thinking, a construct that remains theoretically complex and not directly observable. We conceptualise students' mathematical thinking through the lens of Habermas's theory of rationality (Habermas, 1999), drawing on work that reframes mathematical thinking as rational behaviour manifested in mathematical activity (Morselli & Boero, 2009). From this perspective, students' understanding is treated as observable participation in mathematical practices. Given that thinking is cognitively complex and cannot be directly observed, students' thinking must be examined through their behaviours (Boero, 2006). In this study, we operationalise students' behaviours (Morselli & Boero, 2009) as the ways in which they engage with a mathematical task: (i) justifying the validity of mathematical statements and inferences; (ii) selecting and using appropriate tools; and (iii) communicating their ideas using mathematical terminology and representations appropriate to their

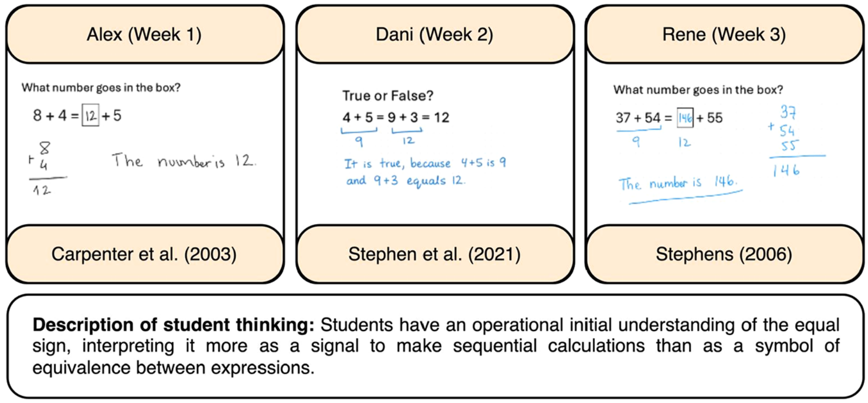


Fig. 2 Simulated Students

grade level and classroom context. We investigate how PTs notice these behaviours in students' mathematical thinking.

In the attending dimension, teachers' attention is directed towards manifestations of students' rational behaviour. Rather than focusing on surface-level features such as correctness, teachers attend to whether students justify their claims, use strategies aligned with task goals, and express their reasoning clearly. This extends van Es and Sherin's (2021) notion of selective attention by specifying both what is attended to and the criteria guiding that attention.

Interpreting involves making sense of these behaviours through teachers' professional knowledge and an inquiry stance. From a rationality perspective, this includes evaluating the validity of justifications, the appropriateness of strategies, and the coherence of communication. This shifts interpretation away from simple correctness judgments towards analysing the quality of students' participation in mathematical practices.

The shaping dimension refers to teachers' active role in constructing interactions that both reveal and develop students' rational behaviour. While van Es and Sherin (2021) conceptualise shaping as creating opportunities to access student thinking, we extend this by emphasising how teachers structure conditions under which rational behaviour becomes visible and can be further developed. This includes prompting justification, supporting strategic choices, and fostering clear communication.

4 Methods

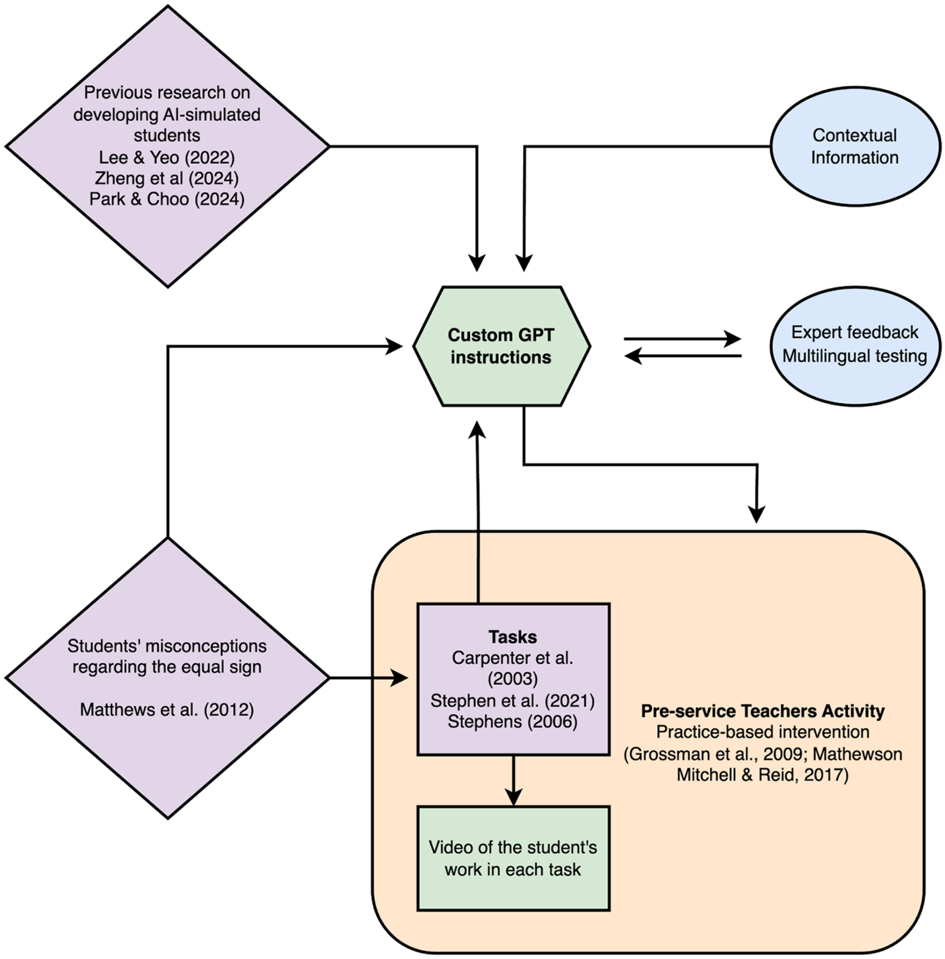
4.1 AI-based simulations construction

Our focus was on creating AI-based simulations for three virtual students, representing a secondary student (aged 11–12 years) with a common misconception regarding the equal sign, and an AI chatbot for a virtual mentor to provide feedback. Each virtual student was paired with a targeted task and a video demonstrating the student's solution. The virtual students were designed to engage in realistic, plausible dialogues, allowing PTs to practise teaching in a controlled environment. Each simulated student embodied a specific misconception. PTs interacted each week with one of the students presented in Fig. 2. We acknowledge that the tasks used may have influenced noticing transitions; we, therefore, consider this as a limitation of the study.

The AI-based simulations were built on OpenAI's GPT-4o (OpenAI, 2024). Following the framework proposed by Park and Choo (2024), we defined each chatbot's persona, aim, recipients, themes and chat structure with examples through explicit prompt engineering statements. Figure 3 illustrates how we design AI-based simulations. For the mentor chatbot, we applied the same framework (Park & Choo, 2024), specifying the feedback strategies to be employed in teacher–student interactions. We provided a few-shot examples to demonstrate how feedback should be given, focusing on concise, situation-specific responses rather than extended explanations. Feedback includes strengths of teachers, aspects to improve and suggestions for improvements.

As researchers, we first tested all prototypes ourselves. Consistent with prior literature, we observed that the student agents sometimes produced overly elaborate responses rather than authentically simulating student reasoning. Similarly, the mentor agent occasionally offered descriptive rather than actionable feedback. These issues were identified and addressed before further testing. Then, a pilot study was conducted in Austria with PTs. One-to-one trials were observed by the researchers, enabling both technical and pedagogical issues to be identified during the process. Based on teachers' perceptions, the prompts were revised. A second pilot study was then carried out in Spanish with in-service teachers. These participants were chosen because of their richer experience of student interactions and their expertise in identifying misconceptions. Their feedback confirmed that the content and structure were pedagogically appropriate. Finally, expert educators in mathematics education trialled the system in Turkish, providing additional perspectives on the implementation. The detailed technical

Fig. 3 Design of AI-based Simulations



design (see Electronic Supplementary Material [ESM]) and development of the AI-based simulations are presented in Tejera et al. (in press).

4.2 Design of the implementation

The implementation was conducted over three weeks, with each week consisting of four phases. Figure 4 provides an overview of the four phases, along with a brief description of the content of each phase.

In the first phase, PTs watched videos presenting student's misconception. Prior studies have frequently employed video materials to support teachers in learning to notice student thinking (Amador et al., 2021). In our study, the videos depicted students' solutions representing the misconception about the equal sign. This method provided PTs with initial evidence of students' reasoning. A total of three student's misconceptions (Fig. 2) aimed to capture a broad range of student thinking and to present PTs with authentic and realistic scenarios.

In the second phase, PTs interacted with the student chatbot, engaging in teaching simulations. This allowed them to probe the student's reasoning further, to elicit additional evidence regarding the student's understanding of the equal sign, and to attempt to overcome the misconception. PTs were not required to complete the simulations within a fixed time; rather, they were free to participate at their own pace and convenience.

The third phase involved reflection on their conversations with the simulated students. PTs analysed the teacher–student interactions by responding to three guiding questions (Fig. 4). To facilitate this process, they were provided with prompts designed to examine their noticing. During this reflection, PTs considered their entire interaction with virtual student, attending to key moments, interpreting the student's understanding, and employing shaping strategies to gain further information and. Each PT was asked to document their reflections by copying their conversation into a separate file and adding comments linked to both their own and the chatbot's responses.

Finally, in the fourth phase, PTs received feedback from the mentor chatbot on their interactions with the student chatbot.

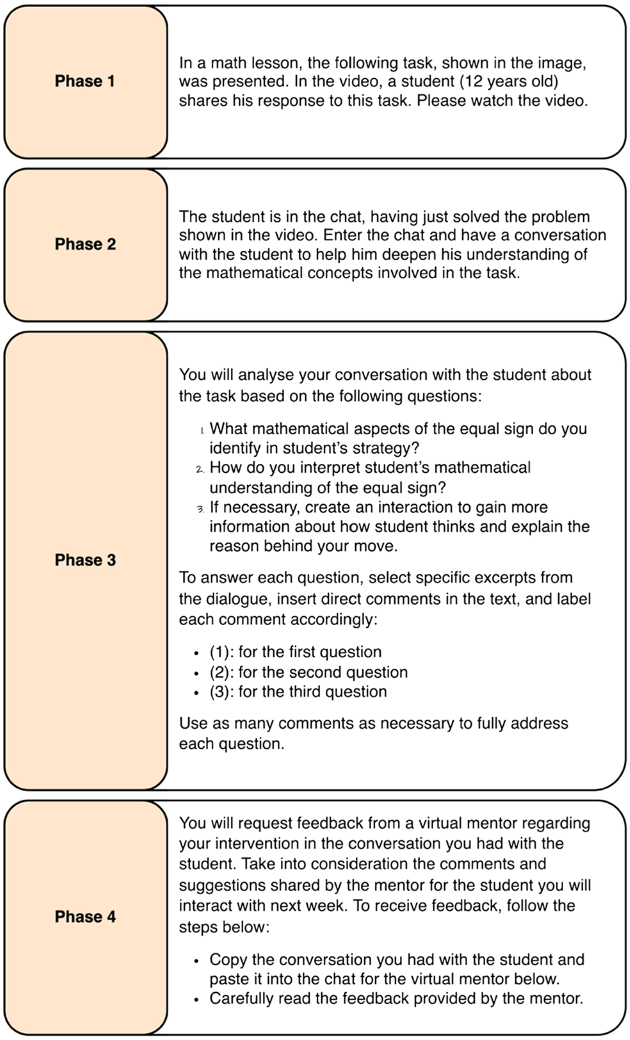


Fig. 4 Overview of the Implementation

4.3 Data collection and participants

We implemented this study in Austria, Türkiye, and Uruguay. In Austria, mathematics teacher education is offered as a bachelor–master pathway within the secondary general track (Bundesministerium für Bildung, Wissenschaft und Forschung, n.d.) , jointly run by universities and university colleges of education; PTs typically combine two teaching subjects, beginning with disciplinary studies and education science foundations and then progressing to subject didactics, classroom management, and assessment, while school-based pedagogical–practical studies are integrated across the entire program and intensify toward the end; a mentored one-year induction phase supports the transition into full teaching. Mathematics teacher education in Türkiye (National Council of Education, 2018) is offered as a four-year undergraduate degree within the faculties of education, where students first complete a content knowledge in mathematics (e.g., calculus, algebra, differential equations, geometry, statistics) alongside introductory courses in educational sciences, and later progress to more specialised courses in teaching methods, classroom management, and assessment, with school-based teaching practice concentrated in the senior year. In Uruguay, it is offered as a four-year concurrent program delivered by the Instituto de Profesores Artigas and regional teacher education centers (Consejo de Formación en Educación, 2023). PTs specialise in mathematics from the beginning, studying algebra, geometry, analysis, and probability alongside general education, psychology, and didactics. The practicum constitutes the backbone of the program, integrating theory and practice from the first year onward. It begins with classroom observation and reflective analysis, advances to planning and partial teaching in the middle years and culminates in a comprehensive supervised teaching practice during the senior year.

In this study, PTs in the final year of undergraduate programs in three countries participated, with the criterion that they had successfully completed their training in subject matter knowledge, pedagogical knowledge, and pedagogical content knowledge, and it was assumed that they possessed these competencies. Furthermore, it was assumed that all PTs had completed and/or were undertaking teaching practices, giving them opportunities to observe classroom situations and student thinking and teach mathematics in a classroom. The study involved a small and uneven international sample, which may limit the generalisability of the findings. Moreover, contextual differences may have influenced how PTs engaged with the simulations and organised their noticing practices. Variations in PTs' backgrounds, therefore, should be considered as a limitation of the study.

Within the scope of the study, approximately 40 PTs were contacted and informed about the project and invited to participate voluntarily by reaching out via email. A total of 24 PTs expressed their willingness to join, and an informational meeting was held with them to explain the purpose and procedures of the study. During the three-week process, seven PTs either submitted incomplete content or failed to deliver the weekly tasks on time, instead submitting all materials toward the final weeks; therefore, their contributions were excluded from the analysis. Seven PTs from Austria, seven from Türkiye, and three from Uruguay were included in the study. We asked PTs about their use of AI and their perceptions of it before starting the study via an online survey. All PTs reported having prior experience with LLMs (e.g., Gemini, ChatGPT) and expressed generally positive attitudes toward their use. They indicated that they mainly used AI chatbots for completing assignments, translation, generating ideas for teaching, identifying student errors, designing classroom activities, and improving lesson plans. Based on their experience, they considered AI chatbots to be useful tools. Since all PTs were already familiar with and experienced in using AI chatbots, we assumed that technical problems would not occur and that the novelty effect would be minimised.

All implementations in each country were conducted in the respective national language. For each country, a Google Classroom was created, and all PTs were invited as participants. Weekly materials were assigned every Monday as tasks, with reminders sent on Wednesdays, and PTs were asked to complete the activities within one week. PTs reported that they completed the study in approximately one hour, with the AI–student interactions taking about 20 minutes and writing the reflections taking around 30 minutes. In cases where PTs encountered technical problems or had questions, they contacted the researcher, who provided support and resolved the issues.

4.4 Data analysis

All data were standardised and anonymised prior to analysis. Materials from 17 PTs were translated into English with the support of ChatGPT and subsequently reviewed by language experts and mathematics educators to ensure linguistic accuracy and appropriate mathematical terminology. The final dataset comprised 17 PTs across three weeks, yielding 51 cases. PTs' noticing was analysed through their reflection papers, in which they commented on their AI student interactions in response to noticing prompts. The unit of analysis was individual noticing statements; a total of 304 statements were identified, each treated as a single analytic unit. To foreground transitions between noticing dimensions, PTs were asked to reflect weekly on their interactions (see Fig. 4). Attending and interpreting were primarily derived from written reflections, whereas shaping was identified from PTs' interaction with accompanying justifications provided in their reflection papers.

A deductive content analysis was employed using an adapted coding scheme. The coding categories were informed by the TNRF (Galiç et al., in press) and related studies (Galiç & Dost, 2025; Galiç et al., 2025) and were adapted prior to coding to align with the construct map of the equal sign (Matthews et al., 2012). Initial coding generated several subcodes within each noticing dimension (e.g., A1–A4 under Attending). These subcodes captured more specific forms of noticing episodes in PTs' interactions. Subcodes were grouped under the broader noticing dimensions which were used as the primary analytic nodes. Figure 5 presents the coding structure, including code labels, descriptions, and illustrative examples.

The codebook was trialled on two randomly selected cases (47 noticing statements), which were coded individually by each author. Discrepancies in coding were resolved through collaborative meetings. Then, ten additional cases (257 noticing statements) were randomly selected and blind-coded by the three authors. Blind coding aligns with best practices to minimise bias (Krippendorff, 2011). Inter-rater reliability was assessed using Krippendorff's α, yielding $\alpha = .803$ on the coder-by-noticing statement matrix (Hayes & Krippendorff, 2007), thereby surpassing the threshold of $\alpha \geq .80$ (Landis & Koch, 1977). Once this reliability level was achieved, we reviewed only the 21 noticing statements where two coders disagreed, following standard reconciliation protocols that prevent excessive negotiation and artificially inflated agreement (Krippendorff, 2011). Each disputed noticing statement was resolved by consensus. The remaining data were then randomly and evenly distributed between two coders for independent coding. Cases 18, 27, 36, 45 and 51 overlapped between the two coders for Cohen's κ calculations at five intervals, allowing us to monitor coder drift (McHugh, 2012). A and B independently applied the finalised codebook (Table 2) for each assigned case, flagging uncertainties. Ambiguities were discussed immediately with C. After each overlapping case, we computed Cohen's κ for a drift check. Whenever it fell below our recalibration threshold of 0.85, which happened after Case 18 ($\kappa = 0.821$) and Case 27 ($\kappa = 0.825$), the three coders met briefly to clarify code definitions and re-code the affected noticing statements. No recalibration was necessary after Cases 36, 45 and 51. Across all overlapping cases, pooled inter-rater reliability was excellent ($\kappa = 0.937$, 95% CI 0.88–0.99). To enhance transparency and credibility, we report the positionality of the three coders. Two coders have expertise in teacher noticing of students' mathematical thinking, while one specialises in qualitative analysis. Although the coders shared a theoretical orientation toward responsive and strength-based teacher education, all coding was conducted using blind procedures to minimise interpretive bias.

The quantitative analysis focused on the frequencies of transitions among noticing dimensions to capture how PTs moved between different noticing dimensions. Following the qualitative coding process, transitions between noticing dimensions were identified by examining the sequential flow of coded segments within each PT's response. Each transition was systematically recorded, and the frequency of these transitions was calculated across all participants. This enabled a descriptive analysis of the distribution of transitions among noticing dimensions based on the tabulation of coded qualitative data (Krippendorff, 2011). To complement this frequency-based analysis, Epistemic Network Analysis (ENA; Shaffer et al., 2016) was employed to model co-occurrences among noticing dimensions as epistemic networks. While the transition counts capture how often PTs moved between dimensions, ENA provides a relational perspective by visualising how these dimensions are interconnected. ENA, therefore, supports and extends the tabulation by revealing co-occurrence among noticing dimensions. Findings are presented on a weekly basis, as different tasks were used each week and were expected to influence noticing transitions.

Dimension	Code Labels	Description	Example
Attending (A)	A1	Attends to students' demonstrated understanding of the equal sign in terms of operational use.	*The student writes the result when he sees the equal sign such as 8 + 4 = 12+5.*
	A2	Attends to students' demonstrated understanding of the equal sign as a relational concept.	*The student shifts from seeing 8 + 4 = 12+5 as a calculation.*
	A3	Attends to students' demonstrated ability to solve equations.	*The student attempts 9+3 = 12 but simply substitutes random numbers without reasoning about equality.*
	A4	Attends to students' demonstrated explanations of the equal sign.	*The student says that equal sign says both side is equal.*
Interpreting (I)	I1	Interprets students' demonstrated understanding of the equal sign by providing supporting evidence.	*The teacher notes that in 4 + 5 = 9, the student interprets "=" only as "do the operation."*
	I2	Interprets students' demonstrated reasoning about the equal sign by providing supporting evidence.	*The teacher infers that the student is moving beyond computation by recognising in 8 + 4 = 7 + 5 that equality compares two sides.*
	I3	Interprets students' demonstrated ability to solve equations by providing supporting evidence.	*With guidance, the student solves in 8 + 4 = 7 + 5 by reasoning rather than just adding.*
	I4	Interprets students' demonstrated explanations of the equal sign by providing supporting evidence.	*The student uses a balance metaphor, explaining "Equal is like a balance. Both side must weigh the same, 12 on one side, 7 + 5 on the other side.*
Shaping (S)	S1	Creates interactions to obtain further information about students' understanding of the equal sign.	*Teacher asked ,"what does the equal sign mean in 8 + 4 = 7+5?"*
	S2	Creates interactions to obtain further information about students' processes of solving equations.	*Teacher asked, "why did you write 12 in the box in □ +5?"*
	S3	Creates interactions to obtain further information about students' ability to explain the equal sign.	*Teacher asked, "how did you know that 8+4 is equal to 7 +5? I saw your operation, but I want you to explain it."*

Fig. 5 Codebook

5 Findings

5.1 Group-level distribution of noticing dimensions

Group-level analyses were conducted using all coded noticing instances from the 17 PTs across the three-week intervention. Figure 6 presents the transitions count among Attending (A), Interpreting (I), and Shaping (S).

Across all three weeks, within-dimension transitions were prominent, indicating that PTs often sustained their engagement within a single noticing dimension during interaction. However, the dominant within-dimension transition varied across weeks. In Week 1, transitions within Attending (A→A) occurred most frequently; in Week 2, transitions within Interpreting (I→I) were most frequent; and in Week 3, transitions within Shaping (S→S) were most frequent.

Transitions from Attending to Shaping (A→S) occurred more frequently than transitions from Attending to Interpreting (A→I) suggesting that PTs more often moved directly from attending to shaping rather than through interpretation. In addition, transitions between Shaping and Interpreting (S→I and I→S) were frequent, highlighting a close relationship between sense-making and gaining more

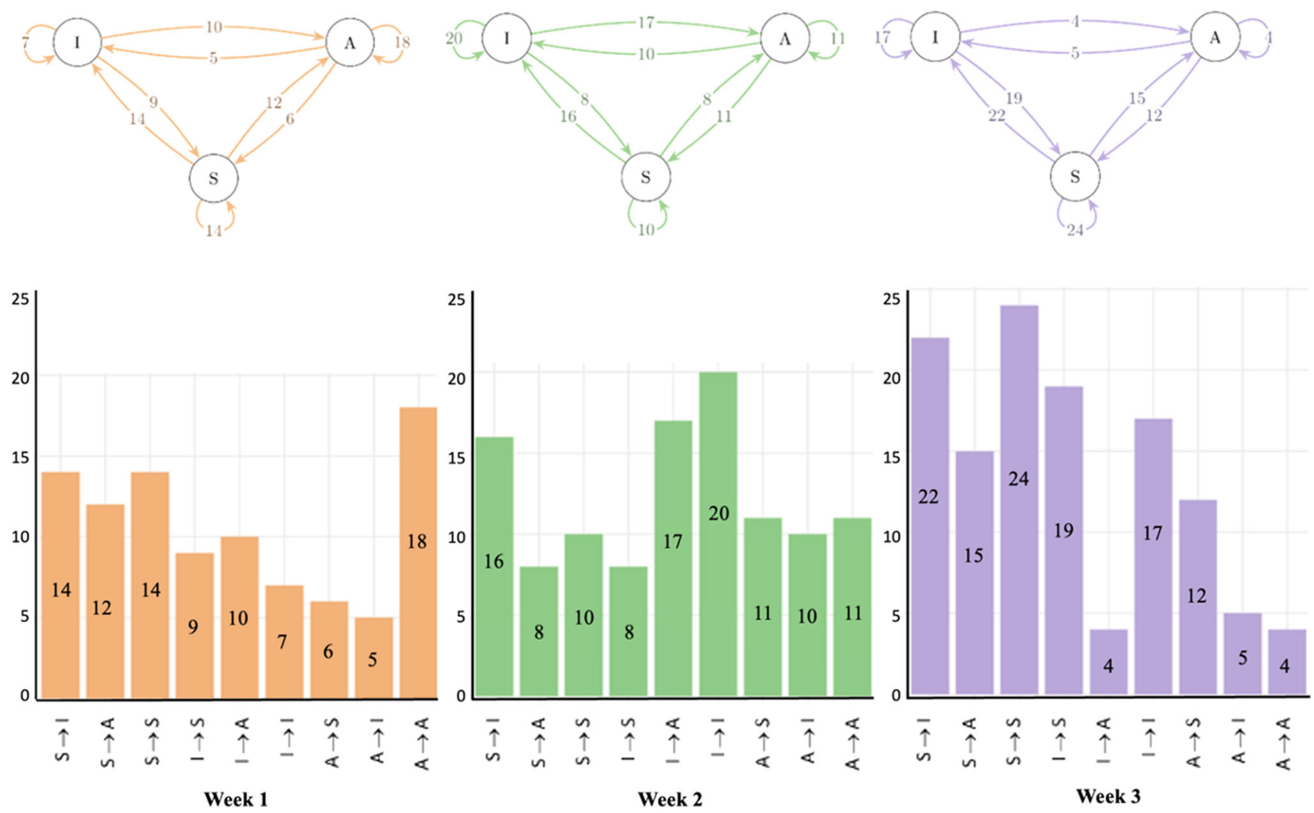


Fig. 6 Transitions Among Noticing Dimensions

details. By contrast, transitions between Attending and Interpreting were comparatively less frequent, indicating that connections between attending and interpreting were less consistently established. Transitions involving Interpreting were also observed across the dataset, reflecting the role of interpretation in coordinating noticing dimensions. The distribution of transitions shows that PTs' noticing was organised through both within-dimension continuity and between-dimension connections.

Figure 7 presents the dominant noticing transition for each PT by week, indicating how many PTs were characterised by each transition in each week.

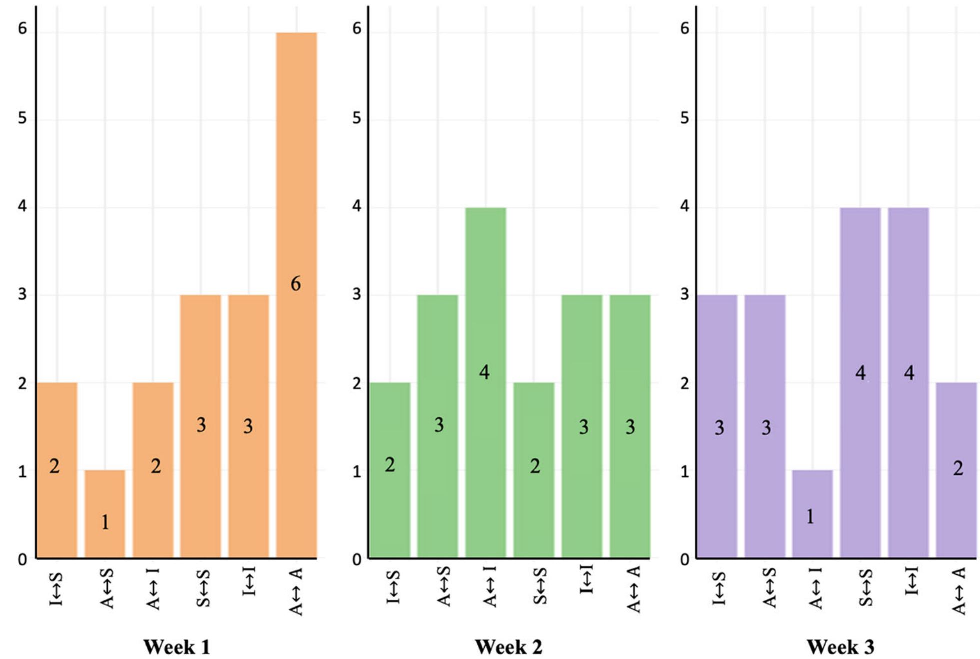
Across the dataset, dominant transitions were distributed across both within-dimension and between-dimension categories, indicating variability in how PTs organised their noticing. In Week 1, within-dimension transitions were prominent, particularly within Attending (six PTs), suggesting that for these participants, noticing sequences were primarily sustained within a single dimension. Within Interpreting and within Shaping were also dominant for several PTs, indicating that some participants consistently coordinated their noticing within these dimensions. Between-dimension transitions were less frequently dominant, although Interpreting–Shaping and Attending–Interpreting were observed for a small number of PTs.

In Week 2, dominant transitions were distributed across a wider range of categories, including both within- and between-dimension transitions. Attending–Interpreting emerged as the most frequently observed dominant transition, while within-dimension transitions and transitions involving Attending–Shaping were also evident. This distribution suggests that PTs' noticing was organised through multiple transition types rather than concentrated within a single dimension.

In Week 3, dominant transitions continued to span both within- and between-dimension categories. Within Shaping and within Interpreting were most frequently dominant, while transitions such as Interpreting–Shaping and Attending–Shaping were also observed across several PTs. This indicates that PTs' noticing was organised in diverse ways, involving both sustained engagement within dimensions and movement between them.

The ENA models (Fig. 8) complement the transition-based analysis by providing a relational perspective on how noticing dimensions are connected at the group level. While transition counts capture how PTs move between dimensions sequentially, ENA visualises co-occurrences, showing which dimensions tend to be activated together within the same segments of interaction. In the ENA models, node size reflects the relative prominence of each noticing dimension,

Fig. 7 PTs' Transitions



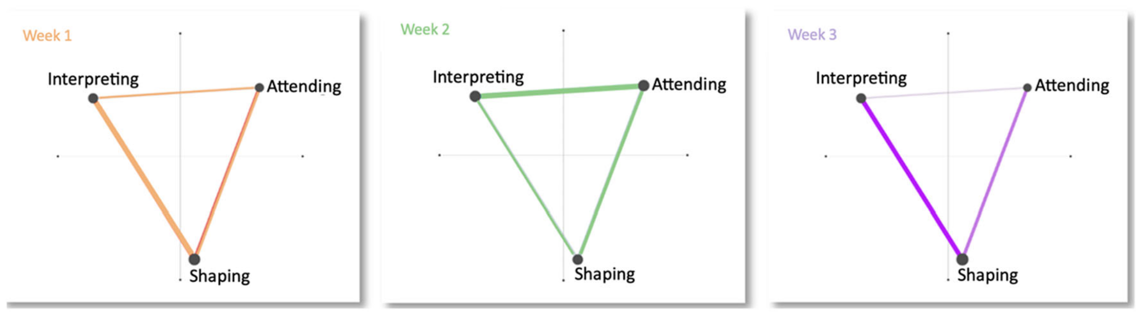


Fig. 8 ENA Across Three Weeks

and edge thickness represents the strength of co-occurrence between dimensions.

Across the three weeks, the strongest co-occurrences were consistently observed between Interpreting and Shaping (Week 1: 0.57; Week 2: 0.54; Week 3: 0.63), indicating a close relationship between sense-making and eliciting. Connections between Attending and Shaping were also present across all three weeks (Week 1: 0.47; Week 2: 0.46; Week 3: 0.46), while Attending–Interpreting links remained comparatively weaker (Week 1: 0.30; Week 2: 0.54; Week 3: 0.23). These results suggest that interpreting is more strongly coordinated with shaping than with attending. The ENA findings align with the transition-based results and extend them by making visible how noticing dimensions are relationally organised within PTs' responses.

5.2 Illustrative case

In Week 1, P4's noticing involved frequent transitions between Interpreting and Shaping, consistent with the dominant group-level transitions among noticing dimensions observed for this week. After initiating the conversation, P4 asked the student to explain the reasoning behind the presented solution and requested additional information to clarify the student's thinking. These interactional moves were accompanied by reflective statements in which P4 articulated interpretations of the student's explanation of the equal sign.

[3] P4 Why did you write 12 in the box in the given problem?
[P4's reflection: I needed to gain a clearer understanding of the student's thinking; therefore, I asked him to explain his reasoning.]

[4] ChatGPT Hmm... Because I added 8 and 4, and the result was 12. That's why I wrote it.
[P4's reflection: The student's knowledge of mathematical equality is directly that an unknown number on either side is equal to the other side of the equation.]

Following the interaction, P4 received mentor feedback structured around strengths, areas for improvement, and suggestions. The feedback emphasised the need to further examine the student's reasoning related to the equal sign and encouraged deeper analysis of the student's explanations.

> [...] greater attention could have been paid to the underlying reasons for the student's misunderstanding. For example, while the explanation "equality means both sides yield the same result" is a useful starting point, the student might have benefited from additional visual or concrete examples to strengthen their understanding. [...]

P4 initially demonstrated a strong instructional intervention regarding the student's understanding of the equal sign; however, the mentor recommended a more in-depth examination of the student's reasoning.

In Week 2, P4's interaction illustrates the prominence of an Attending–Interpreting transition. The virtual student articulated an understanding of the equal sign as requiring equivalence between both sides of the equation. P4 attended to this emerging understanding and interpreted its implications for the student's conceptual grasp, while shaping moves remained present but less dominant.

[12] ChatGPT Hmm, I think I understand... So, the equal sign is saying both sides are the same, right? But here, 4 + 5 is 9, and 9 isn't the same as 9 + 3. So that's not correct, is it? But then is it wrong to think about doing the operations step by step?
[P4's reflection: The student has filled in the gap in knowledge about the equal sign in the sense we gave above in this answer, but he still does not realise that it is not an expression that gives the order of operations.]

[13] P4 First do the operations in sequence, then discuss whether the equal sign really indicates equality between them. What do you think?

[14] ChatGPT $4 + 5 = 9$ and $9 + 3 = 12$. But now, the equal sign doesn't seem to indicate equality, because 9 isn't equal to 12.
[P4's reflection: The student slowly started to understand that the equal sign means equality.]

In Week 3, P4's interaction again reflected an Interpreting–Shaping transition. P4 frequently prompted the student to articulate their thinking, a practice likely influenced by mentor feedback.

[14] ChatGPT There was addition in the question, so I thought I had to add them all.
[P4's reflection: The student showed that he had no knowledge of equality and that applying only the existing operations in expressions containing equality would yield the unknown.]

[15] P4 What do you think about the equal sign? What effect does it have between the sums?
[P4's reflection: I asked this question to enable him to explore the existence of the equal sign and its possible effect on the expression.]

P4's case illustrates how relational transitions among noticing dimensions identified at the group level were instantiated within individual interactions.

6 Discussion and conclusion

This study set out to explore teacher noticing as a dynamic process by analysing transitions among noticing dimensions within AI-based simulations focused on students' understanding of the equal sign. A central contribution of this study lies in foregrounding transitions among noticing dimensions as a way to understand how noticing is organised in AI-based simulations. While prior research has conceptualised noticing as involving interrelated processes (Jacobs et al., 2010), empirical work has rarely captured how teachers move between these dimensions in practice. Recent studies have highlighted the difficulty of empirically capturing these relationships (Thomas et al., 2023). Our findings address this gap by demonstrating that transitions provide a productive unit of analysis for examining noticing. The prominence of transitions between interpreting and shaping highlights the close coupling between sense-making and gaining details. This aligns with perspectives that conceptualise noticing as dynamic, where component processes interact in recursive and reflexive ways rather than following a fixed sequence (Thomas et al., 2023). By making these relationships visible as directional transitions, this study offers a more fine-grained account of how noticing unfolds during interactions with LLMs.

The reduced complexity of the AI-based simulations should be considered. Compared to classroom or video-based contexts, where multiple events compete for attention, simulations focus attention on a single student interaction (Castro Superfine et al., 2026). Prior research has shown that different representations of practice afford different forms of noticing (Dreher & Kuntze, 2015). Thus, the transitions observed in this study may reflect both the affordances and the constraints of AI-based environments. interaction (Castro Superfine et al., 2026). Prior research has shown that For example, van Es and Sherin (2026) conceptualises shaping to include practices such as "leveraging participant structures" and "creating and using material and social resources". These forms of shaping, which rely on the organisation of classroom interactions and the use of physical and social resources, were not observable in the present study due to the text-based and individual nature of the AI-based simulation environment. As a result, shaping in this study was primarily enacted through discursive moves, such as questioning and prompting. The prominence of shaping can be considered in relation to the interactional structure of the simulations in this study. The design afforded repeated opportunities for dialogic engagement focused on students' mathematical reasoning, supporting inquiry into student thinking (Lee & Yeo, 2022; Son et al., 2024).

The findings suggest that noticing operated as an ongoing and recursive process (Thomas et al., 2023) in which PTs' interpretations frequently informed subsequent shaping moves, which in turn created additional opportunities to attend to and interpret students' thinking (van Es & Sherin, 2021). Within the AI-based simulations, PTs were not only responding to students' ideas but also actively influencing how the interaction unfolded through probing, questioning, and eliciting further explanations. This highlights the potential of AI-based environments to support noticing as an interactionally constructed process. The observed movement across noticing dimensions suggests that PTs' noticing did not operate as a linear sequence, but rather as a dynamically coordinated and iterative process shaped through interaction (Thomas et al., 2023). Transitions between attending, interpreting, and shaping indicate that noticing developed through ongoing cycles of sense-making and instructional action. In line with perspectives emphasising the situated and interactional nature of noticing (Jessup, 2023), examining these transitions within AI-based simulations provides insight into how noticing evolved moment-by-moment as PTs interacted with and responded to student thinking.

Noticing is shaped by interactional conditions and task characteristics (Jessup, 2023). Findings reflect context-sensitive ways in which PTs organised their noticing in response to interactional affordances. Differences in transitions across weeks may reflect the influence of task design and interactional conditions, while also indicating how PTs reorganised their noticing in response to these conditions in AI-based simulations. Although all three tasks involved operational interpretations of the equal sign, the comparison between the two expressions was made more explicit in the second task, which may have supported stronger attending–interpreting transitions. Thus, variation across weeks may reflect the context-sensitive and iterative organisation of noticing rather than inconsistency in PTs' noticing practices. This interpretation aligns with research emphasising the interconnected and mutually influential nature of noticing dimensions (Thomas et al., 2023) and with perspectives that conceptualise noticing as theme-specificity (Friesen & Kuntze, 2021; Dreher & Kuntze, 2015).

The findings indicate that AI-based simulations hold promise for supporting teacher noticing in mathematics teacher education. By providing controlled, low-risk environments without ethical concerns related to classroom experimentation, such simulations allow PTs to practise noticing and interaction in focused contexts. Moreover, the use of LLMs as both student and mentor enables fine-grained analysis of noticing processes that may be difficult to capture in real classroom settings. Furthermore, Scheiner and Amador (2025) argued that teachers' proactive role in shaping what becomes noticeable through interactional and contextual design remains an important yet underexplored dimension of teacher noticing. In this regard, we suggest that AI-based simulations may serve as a powerful tool for making visible how PTs decide to respond to students' thinking and how they shape interactional opportunities during noticing processes. AI-based environments may also offer opportunities for PTs to explore how interactions, questioning strategies, and instructional conditions can be designed or redesigned to create richer opportunities for noticing students' thinking. In this sense, AI-based simulations may support alternative approaches to learning to notice that foreground the design of noticing opportunities. These findings align with prior research highlighting the potential of AI-based simulations to support teacher noticing (Bastian et al., 2025; Galiç et al., 2025).

This study forms part of a larger research project examining different aspects of AI-based simulations in teacher education. While the present study focuses specifically on transitions among teacher noticing dimensions, other components of this project address complementary aspects, such as PTs' perceptions of AI-based simulations (Galiç et al., 2026) and the technical design of the AI environment (Tejera et al., in press). In addition, the current study does not systematically analyse the content or uptake of mentor feedback. We, therefore, explore how PTs interpret and respond to feedback, as well as how these processes relate to the development of teacher noticing and teaching moves in the future studies.

The deficit-oriented simulation design, which centred on student misconceptions, likely oriented PTs' noticing toward error detection and corrective reasoning. While this supported focused engagement with conceptual difficulties, it may have constrained opportunities for strength-based noticing (Scheiner, 2026). Future research should explore simulation designs that balance deficit- and strength-based representations of student thinking. Moreover, the text-based nature of the simulations also limited engagement with the multimodal dimensions of mathematical reasoning. Noticing involves coordinating verbal explanations with visual representations, gestures, and symbolic forms in classroom interaction. While text-based environments support interpretive and inquiry-oriented aspects of noticing, they underrepresent perceptual and representational demands central to mathematics teaching. Finally, the short temporal scope of the study and the multilingual implementation introduce additional limitations. The three-week design does not permit conclusions about the durability of noticing practices, and interactions conducted in different languages may have influenced the specificity of AI-generated responses.

This study contributes to mathematics teacher education by demonstrating how AI-based simulations can make the dynamic coordination of noticing dimensions visible, offering a process-oriented approach to understanding and supporting teacher noticing.

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Declarations

Competing Interests The authors declare no competing interests.

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