INTERPRETING STUDENTS' MATHEMATICAL THINKING INVOLVING NATURAL NUMBER BIAS

Sebastián Parodi

Universidad Tecnológica del Uruguay

Natural number bias (NNB) has been widely documented as a persistent difficulty in learning rational numbers. However, less is known about how prospective mathematics teachers (PMTs) interpret student responses, in which this bias becomes evident when working with fractions and decimals. This paper examines how PMTs reconstruct the reasoning underlying such responses by asking: What characterizes PMTs' interpretations of student productions involving manifestations of NNB?

The study draws on two complementary frameworks. First, professional noticing (PN), focusing on interpretation as inferring students' mathematical understanding from the details of their productions and from knowledge of mathematical thinking (Jacobs et al., 2010). Second, conceptual change research provides the basis for understanding students' NNB as a persistent reasoning framework arising from the overgeneralization of natural number properties to rational numbers (Van Dooren et al., 2015).

Nine PMTs participated in a qualitative exploratory case study within a teacher education course. The study was based on a sequence of two teacher-role concept cartoons (TRCC) whose characters are PMTs discussing authentic student productions involving NNB. Data consisted of audio-recorded plenary discussions. Segments containing explicit inferences about students' thinking were analysed and coded into three levels (Jacobs et al., 2010): lack, limited, and robust evidence (LaE, LiE, RoE).

Analysis of the data showed the levels to be characterized as follows. At LaE, errors were reduced to procedural shortcomings; NNB was explicitly ruled out. At LiE, errors were attributed to the inappropriate transfer of learned rules or analogies, with NNB implicitly suggested but not conceptualized as an underlying framework. At RoE, errors were interpreted as coherent results of reasoning with rational numbers as if they were natural numbers, reconstructing NNB as an overgeneralization of natural number properties. These levels capture qualitative differences in interpretive depth rather than developmental stages, positioning TRCC as a resource for inquiring into PMTs' noticing of NNB in students' mathematical thinking.

References

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Van Dooren, W., Lehtinen, E., & Verschaffel, L. (2015). Unraveling the gap between natural and rational numbers. *Learning and Instruction, 37*, 1–4.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 592. PME.